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Detection of Process Control Shifts Using Shewhart (X-Bar) and Cumulative Sum (CUSUM) Charts

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Abstract: *This study compares how the Shewhart (X-Bar) chart detects small, difficult-to-identify shifts and variations in process control, as the Cumulative Sum (CUSUM) chart does. The production process of bread in a Bakery in Gboko, Benue State, Nigeria was studied for four months (August, September, October and November). Data were collected on the daily weights (kg) of shortening fat systems in every mixture in four shifts daily; the Shewhart (X-Bar) chart was used to ascertain whether the process was within or out-of-control, i.e., whether the quantity of shortening fat fused into the production process daily was adequate or not for the bread produced. It was discovered that the production process most consistently met quality standards for shortening weights in November, followed by August, then September, and least effectively in October; based on the number of out-of-control days that the CUSUM chart was able to capture, which the Shewart chart was unable to capture. It was concluded that the CUSUM chart indeed facilitates the detection of small variations that are not typically identified by the X-bar chart.*

Keyword: *Shewarts chart, CUSUM chart, Assignable causes of variation, Shift detection, X-bar chart, Shortening fat, Production process.*

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1.0 Introduction

Cumulative Sum (CUSUM) charts serve as an alternative to Shewhart control charts (X-bar charts). Although Shewhart control charts are widely used and their control violations are well documented, they may be insensitive to certain process shifts. CUSUM charts calculate the average value for subgroups and assess the expected variability around the mean (Nightingale, M. J., et al.,2012). These charts are particularly effective at detecting distinct, steady, and intermittent shifts in a process, and are especially suited for identifying small, sustained shifts. CUSUM charts measure the cumulative deviation from the mean or a specified target value. Depending on the test applied, the chart displays either the standardized deviation from the target or the mean value of the subgroup. Common tests for evaluating out-of-control conditions include the run-sum and the V-mask. On a CUSUM chart, this typically results in a straight line or random fluctuations around the target with a mean of zero. The run-sum method plots the subgroup average, similar to a Shewhart chart. However, rather than applying traditional control chart rules, a user-defined limit or score for cumulative deviation from the target signals a violation. For instance, if the first subgroup average is one unit above the target, the score is set to one. If the next subgroup average is one unit below the target, the score is –1, resulting in a cumulative sum of zero. This outcome aligns with expectations for random variation around the target. If the process mean shifts upward or downward, the cumulative score will continue to increase or decrease. A violation is identified when the cumulative score surpasses a user-defined threshold.

Interpreting a control chart means looking at data points compared to the centerline, upper limit, and lower limit, and spotting patterns or trends that might need fixing. When a process is stable, the chart shows random ups and downs around the centerline, with all points inside the limits. If any points go outside the limits, the process is out of control, showing unexpected changes that need fixing. Even if all points are inside the limits, some patterns can still show problems. Six or more points in a row going up or down, even inside the limits, may mean the process is changing. Also, eight or more points all above or all below the centerline, or runs like ten out of eleven or twelve out of fourteen points on one side, suggest a shift and need fixing. Fourteen or more points in a row that go up then down repeatedly may also show a change. Other warning signs include two out of three points close to the outer edge of the chart (more than two standard deviations from the centerline), four out of five points near the outer middle area (more than one standard deviation from the centerline), and eight points in a row with none close to the centerline, spread on both sides. These patterns all suggest a possible problem that should be checked. Another warning sign is fifteen data points in a row all close to the centerline, no matter which side, which may also show a possible problem in the process. (Aimee Birdsall & Valerie Watts, 2025).

Professional baker recipes are stated using a notation called baker's percentage. The amount of flour is denoted as 100%, and the amounts of the other ingredients are expressed as a percentage of that amount by weight. Measurement by weight is more consistent and accurate than measurement by volume, particularly for dry ingredients. There are various sources of bread that undergo some kind of preparatory treatment or baking process to attain specific demand or quality. However, bread-producing companies are being sealed up by Nigerian Food Monitoring Agencies on account of non-conformance to standards (failure to meet the production requirement). Based on previous experience, this research wishes to use statistical techniques to assess the process quality of bread production at a factory in Gboko and check the extent of shifts recorded in the process using data collected. The production process from a Bread production company is analyzed in this study for variables such as: the weight of powdered flour in kilograms (kg), sugar in kilograms (kg), butter in kilograms (kg), yeast in grams (g), preservative in grams (g), and water in liters. Evidence from 30,000 years ago in Europe revealed starch residue on rocks used for pounding plants; during this time, starch extract from the roots of plants such as cattails and ferns was spread on a flat rock, placed over a fire, and cooked into a primitive form of flat bread. Yeast spores are ubiquitous, including the surface of cereal grains, so any dough left to rest will become naturally leavened (The concept "variation in quality" cannot be ignored if the objective of statistical quality is to be realized. It is possible that two identical products produced one after the other, on the same machine, under the same conditions, and from the same batch of raw materials may vary in size and appearance. This may be due to chance variation or as a result of assignable variation (Moses, 2007). In his study, consideration is given to chance variation, otherwise known as random variation; this is an inherent and inevitable variation in any production process. It cannot be controlled or eliminated totally, and the production process is said to be in a state of statistical control if it is confined to chance variation only. Whereas Assignable variation is relatively large variation that results from special causes and can be eliminated. He stressed that one or more of the following may cause this variation. Poorly trained operation, Raw materials of inferior quality, Raw materials from different batches and varying quality, Faulty machine settings, Unqualified operators, etc. From the reasons listed above, a systematic method of detecting serious deviation from a state of statistical control is needed. Therefore, control charts are designed for this purpose.

Statistical process control is a systematic method for analyzing data (quality characteristics) by monitoring and studying process variation. The goal is to stabilize the process and reduce process variation. The ultimate goal is continuous process improvement. We often use statistical process control (SPC) to monitor and improve manufacturing processes. However, SPC is also commonly used to improve service quality (Bruce and Richard, 2001).In their study, they explained that, before the widespread use of SPC, quality control was based on an inspection approach. Here the product is first made, and then the final product is inspected to eliminate defective items. This is called action on the output of the process. The emphasis here is on detecting defective products that have already been produced. This is costly and wasteful because, if a defective product is produced, the bad items must be scrapped, reworked, or reprocessed (that is, fixed) or Downgraded (sold off at a lower price). In contrast to the inspection approach, SPC emphasizes integrating quality improvement into the process. Here, the goal is to prevent bad quality by taking appropriate action on the process.

2.0 Materials and Methods

Conventional statistical process control (SPC) charting is an established monitoring and diagnostic method currently being developed across various healthcare monitoring applications. The use of risk-adjusted (RA) control charts to investigate clinical binary outcomes represents a significant area of research. Previous studies have extended the monitoring of binary outcomes in cardiac surgeries by applying logistic models to estimate patient death probabilities based on individual risk profiles. Consequently, several RA-based cumulative sum (CUSUM) charts have been proposed for tracking 30-day patient mortality. In this context, a novel run rules method is introduced alongside the RA CUSUM control chart. The proposed approach was evaluated and benchmarked using simulation studies with the average run length (ARL) metric. Results were favorable, and additional analysis under beta-distributed conditions further demonstrated the method's robustness (Hussain, Z. et al. 2025).

A standard bread formulation typically includes approximately 3 percent shortening by flour weight. Excessive fat content can inhibit dough rise during proofing. Bread shortening serves as a lubricant during dough mixing. Both shortenings and margarines are fat systems designed to provide specific nutritional and functional properties for various applications. Shortenings generally contain 100 percent fat, whereas margarines contain about 80 percent fat (see fact sheet FAPC-210). Shortenings are derived from vegetable oils and animal fats. Traditional products such as lard, tallow, and ghee consist entirely of animal fat. Vanaspati, a vegetable oil-based product with properties similar to shortening, is widely used in Eastern countries. The term "shortening" denotes the ability to impart "shortness" to food products. Shortenings function by lubricating and weakening the structure of food components, thereby providing desirable textural properties in the final product. In baked goods prepared without shortening, gluten and starch particles adhere, resulting in a hard and tough texture. Conversely, the inclusion of shortening disrupts the continuity of protein and starch structures, producing a tender, flaky, and aerated bakery product. During frying, shortenings enable rapid and uniform heat transfer and form a moisture barrier that retains moisture within the food, resulting in a moist final product. Typically, shortenings are soft, odorless, bland in flavor, and exhibit a plastic-solid consistency. The term plastic describes a solid, non-fluid, non-pourable, and non-pumpable state at room temperature. Additionally, shortenings are available in liquid, dry powder, pellet, or flake forms, and are often encapsulated in water-soluble materials such as skim milk, cheese whey, or soy protein isolate (Robert M. Kerr Food & Agricultural Products Center 2017).

2.1 Control Charts-$\bar{X}$- chart

Control charts are commonly used to monitor the quality of a product in an ongoing manufacturing process. They allow quality control experts to closely observe variation in the process and alert manufacturers to changes in the product. Keller (2003) said that a control chart is one of the most commonly used methods of statistical process control (SPC), which monitors the stability of the process. The main components of a control chart include the central line and control limits (UCL); below the central line is the Lower Control Limit (LCL). This is achieved by plotting the data obtained from samples taken periodically at regular intervals. It is possible to check by means of such a chart whether the variation between samples may be attributed to chance or whether a problem has entered the process. The upper and lower control limits are utilized as the decision criteria. When the sample point falls beyond them, one looks for a problem, that is, for sources of assignable variation; otherwise, the process is allowed to continue.For an $\bar{X}$-chart, the test statistics is the sample mean $\bar{X}$. That is, we compute the mean for each of a continuing series of sample taken. For each series of sample, we want to determine whether there is sufficient evidence to infer that the process mean has changed. We reject the null hypothesis if at any time the sample mean falls outside the control limits (Keller & Gerald 2003)

2.2 Cummulative Sum Chart

A CUSUM chart is a time-weighted control chart that displays the cumulative sums (CUSUMs) of the deviations of each sample value from the target value. Because it is cumulative, even minor drifting in the process mean will lead to steadily increasing or decreasing cumulative deviation values. Therefore, this chart is especially useful in detecting slow shifts away from the target value due to machine wear, calibration problems, and so on. If a trend develops upward or downward, it indicates that the process mean has shifted, and you should look for special causes (Applying CUSUM to hockey prediction models, 2016). The plot points can be based on either subgroups or individual observations. When data are in subgroups, the mean of all the observations in each subgroup is calculated. CUSUM statistics are then calculated from these means. When you have individual observations, CUSUM statistics are calculated from the individual observations.

2.3 Data Quality Assurance

Although the dataset used in this study is secondary data, assurance was given that several prior measures were used to control data quality throughout the four months when data for the study were collected, including using skilled data collectors working in the factory. Seasoned supervisors/team leads undertook proper calibrations of the weighing scales used before proceeding to collect the data at every shift (mixing process); close monitoring probing for digit preference was also used.

2.4 Source of Data

The data used in this research is secondary data collected from the quality control laboratory of a bread production company in Gkoko, Benue State Nigeria. The data collected is the **content of shortening (fats) in kilograms (kg)** which is one of the raw materials used in the production of bread and it covers a period of four (4) months (August, September, October and November) in which a series of samples were taken to analyze the data- usually called subgroup over time

2.5 Data Processing and Statistical Analysis

One of the well-known tools of statistical process control is the control chart, including the mean chart or the X-bar Chart ($\bar{X}$ −chart) which is also known as Shewhart charts. Control chart is a graphical tool based on the measurement of data obtained in the course of time from the process. Based on the nature of the data obtained from the process, both the Shewhart and Cumulative SUM charts (CUSUM) will be used and compared, to make that sure that small sustained shifts in the process are detected. The Quality Control menu in the SPSS software is used with corroboration from Minitab in the analysis of the data collected in this study.

2.6.1 Means charts ($\bar{X}$ -chart)

This is a control chart that monitors the means of the sample draw from a population. It shows variation in the average of samples. The means is obtained by summing up all the measurement and dividing by the number of the observation (Ekpo, A., 2016).

$$\bar{X} = \frac{X_1 + X_2 + \dots X_n}{n} = \sum_{i=1}^{n} X_i / n \qquad (1)$$

where $\sum_{i=1}^{n} X_i$ = sum of all the measurements in the sample of size n (i = individual sample, 1, 2, …, 30; n = 4), $\bar{X} \sim N(\mu, \delta^2)$ is the sample mean following Z-normal distribution with μ and mean of the variable δ^2/n. The E(x) = μ and variation $(x) = \delta^2$ (2)

Then $UCL = \bar{\bar{X}} + Z\sigma_{\bar{x}}$. (3)

$LCL = \bar{\bar{X}} - Z\sigma_{\bar{x}}$ (4)

Where UCL = Upper Control Limit, CL = Central Line, LCL = Lower Control Limit, $\bar{\bar{X}}$ = Average of the sample mean

Z = Standard normal variation (5)

When $\sigma_{\bar{x}}$ = Standard Deviation of the distribution of the samples means, computed as:

$\frac{\sigma}{\sqrt{n}}$, with σ as the process standard deviation

n - Sample size (number of observations per samples)

Where $2\sigma \rightarrow$ 2 Sigma =95.44% CI (6)

$3\sigma \rightarrow$ 2 Sigma =99.74% CI (7)

$UCL\bar{x} = \bar{\bar{X}} + A_2\bar{R}$ (8)

$LCL\bar{x} = \bar{\bar{X}} - A_2\bar{R}$ (9)

Where A_2 = is a constant value read from statistical table.

The mean range is given as equation 10

$$(\bar{R}) = \sum_{j=1}^{k} R_j / k = \frac{R_1 + R_2 + R_3 + \dots R_k}{k} \qquad (10)$$

2.6.2: The Cumulative Sum Chart (CUSUM Chart)

In statistical process control (SPC), because Shewart's chart has deficiencies, the CUSUM chart is used to capture small shifts that Shewart's chart cannot (Ekpo, A., 2016). The CUSUM chart is given by:

$S_n = \sum_{i=1}^{n}(x_i - \mu) + c_i$ (11)

Where, x_i = result from individual sample data

μ = mean of the 1st and 2nd 10 samples

$C_i = (x_i - \mu) + [x_i - \mu + (x_i - \mu)_{i-1}]$ (12)

3.0 Results and Discussion

The results presented here, is based on the four shifts (X_1-X_4) sampled weights in kg. of shortenings collected daily in a Bread production factory in Gboko for four months (August -November).

3.1 Analysis on weight of shortening fused in production of bread in (kg) in the month of August using the Mean charts ($\bar{X}$ -chart)

Table 1 presents the daily measurements of shortening (kg) used in bread production during the month of August, while Fig. 1 illustrates the corresponding Shewhart Mean ($\bar{X}$) control chart. The daily sample means ranged from 11.8 to 12.2 kg, indicating relatively small variations in the quantity of shortening used throughout the production period. Similarly, the observed sample ranges varied between 0.0 and 0.8 kg, suggesting that the within-sample variability remained consistently low. As shown in Fig. 1, all 31 sample means lie within the established Upper Control Limit (UCL) and Lower Control Limit (LCL), with no points exceeding the control boundaries. Furthermore, the plotted points fluctuate randomly around the centre line without exhibiting prolonged upward or downward trends, cyclic patterns, or systematic clustering.

Table 1: Weight of shortening used in production of bread in (kg) in the months of August

S/No.	Daily Readings					
	X1	X2	X3	X4	Mean ($\overline{X}$)	Range
1	11.5	11.7	11.8	12.1	11.8	0.6
2	12.0	12.4	11.9	12.0	12.1	0.5
3	12.2	12.3	12.3	12.3	12.2	0.0
4	11.7	12.1	11.8	11.6	11.8	0.5
5	12.1	11.9	11.9	12.2	12.0	0.3
6	12.3	12.0	11.8	12.3	12.1	0.5
7	12.4	12.2	12.1	11.8	12.1	06
8	12.1	12.4	12.3	12.0	12.2	0.4
9	11.8	11.9	11.8	11.8	11.8	0.1
10	11.6	12.1	11.9	12.0	11.9	0.5
11	12.0	11.8	12.2	12.4	12.1	0.6
12	11.9	12.3	11.8	12.1	12.0	0.5
13	12.2	12.0	12.3	12.2	12.2	0.3
14	11.8	11.6	12.0	11.9	11.8	0.4
15	12.4	12.5	11.7	11.9	12.1	0.8
16	12.3	12.0	11.8	11.6	11.9	0.7
17	11.8	12.3	12.1	11.8	12.0	0.5
18	12.3	11.6	12.0	12.4	12.1	0.8
19	11.6	12.0	12.2	12.3	12.0	0.7
20	11.8	11.9	12.0	12.2	12.0	0.4
21	12.1	11.7	12.3	11.9	12.0	0.6
22	12.1	11.7	12.4	11.9	12.0	0.7
23	11.8	12.0	12.1	11.7	11.9	0.4
24	12.2	12.2	12.0	11.8	12.1	0.6
25	12.0	11.7	11.8	12.1	11.9	0.4
26	11.8	12.3	12.0	11.7	12.0	0.6
27	12.2	12.4	11.9	12.3	12.2	0.5
28	11.7	11.8	11.9	11.6	11.8	0.3
29	11.8	11.7	11.8	12.1	11.9	0.4
30	12.1	11.6	12.4	11.7	12.1	0.8
31	11.6	12.1	11.9	12.0	11.9	0.5

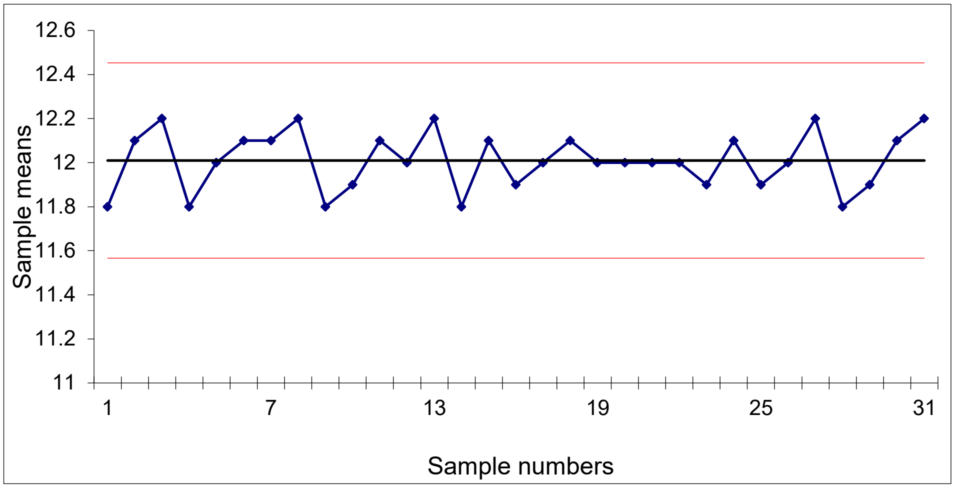

Fig. 1: Shewart Chart showing in and out of control variables in August

These characteristics indicate that the shortening dispensing process was statistically stable throughout the month of August and that the observed fluctuations resulted primarily from common-cause variation inherent in routine production operations rather than assignable causes.

The absence of out-of-control observations in Fig. 1 is further supported by the descriptive statistics in Table 1, where the sample means remained tightly concentrated around the process average of approximately 12.0 kg. Although isolated observations recorded means of 11.8 kg (e.g., Samples 1, 4, 9, 14 and 28) and 12.2 kg (e.g., Samples 3, 13 and 27), these values remained well within the statistical control limits and therefore do not indicate process instability. Likewise, samples exhibiting relatively larger ranges (0.8 kg) did not correspond to abnormal shifts in the process mean, suggesting that short-term variability did not adversely affect the overall process performance.

The stability observed in the $\bar{X}$ chart implies that the production process maintained consistent control over shortening usage despite normal day-to-day operational variations. Such consistency is essential in commercial bread production because shortening directly influences dough handling characteristics, loaf volume, crumb softness, shelf life, and overall product quality. Maintaining the ingredient quantity within statistically acceptable limits minimizes formulation inconsistencies and contributes to uniform product characteristics across production batches. These findings are consistent with the principles of Statistical Process Control (SPC), which state that a process is considered to be in statistical control when all observations fall within the control limits and exhibit random variation without violating established control rules (Aslam et al., 2021; Hussain et al., 2025). Similar observations have been reported by Nightingale et al. (2012), who demonstrated that Shewhart control charts effectively distinguish between common-cause and special-cause variations in quality monitoring systems. The present findings therefore suggest that the bakery's shortening dispensing process was operating efficiently during August, with no evidence of process deterioration or the need for corrective intervention.

3.2 *Analysis on weight of shortening fused in production of bread in (kg) in the month of August using the CUSUM-Char*

Table 2 presents the cumulative sum (CUSUM) statistics computed from the daily mean weights of shortening used in bread production during the month of August, while Fig. 2 illustrates the corresponding CUSUM control chart. The CUSUM values were obtained by cumulatively summing the deviations of the daily sample means from the target process mean of 12.01 kg. Unlike the Shewhart $\bar{X}$ chart, which is primarily designed to detect large process shifts, the CUSUM chart accumulates small deviations over time, making it considerably more sensitive to subtle changes in process performance.

Table 2: Weight of shortening fused in production of bread in (kg) in the month of August

Sample, i	X_i	$(X_i – 12.01)$	$C_i = (X_i – 12.01) + C_i - 1$
1	11.8	-0.21	-0.21
2	12.1	0.09	-0.12
3	12.2	0.19	0.07
4	11.8	-0.21	-0.41
5	12.0	-0.01	-0.15
6	12.1	0.09	-0.06
7	12.1	0.09	0.03
8	12.2	0.19	0.22
9	11.8	-0.21	0.01
10	11.9	-0.11	-0.1
11	12.1	0.09	-0.01
12	12.0	-0.01	-0.02
13	12.2	0.19	0.07
14	11.8	-0.21	-0.14
15	12.1	0.09	-0.05
16	11.9	-0.11	-0.16
17	12.0	-0.01	0.17
18	12.1	0.09	-0.08
19	12.0	-0.01	-0.09
20	12.0	-0.01	-0.1
21	12.0	-0.01	-0.11
22	12.0	-0.01	-0.12
23	11.9	-0.11	-0.23
24	12.1	0.09	-0.4
25	11.9	-0.11	-0.25
26	12.0	-0.01	-0.26
27	12.2	0.19	-0.07
28	11.8	-0.21	-0.28
29	11.9	-0.11	-0.39
30	12.1	-0.09	-0.3
31	11.9	-0.11	-0.41

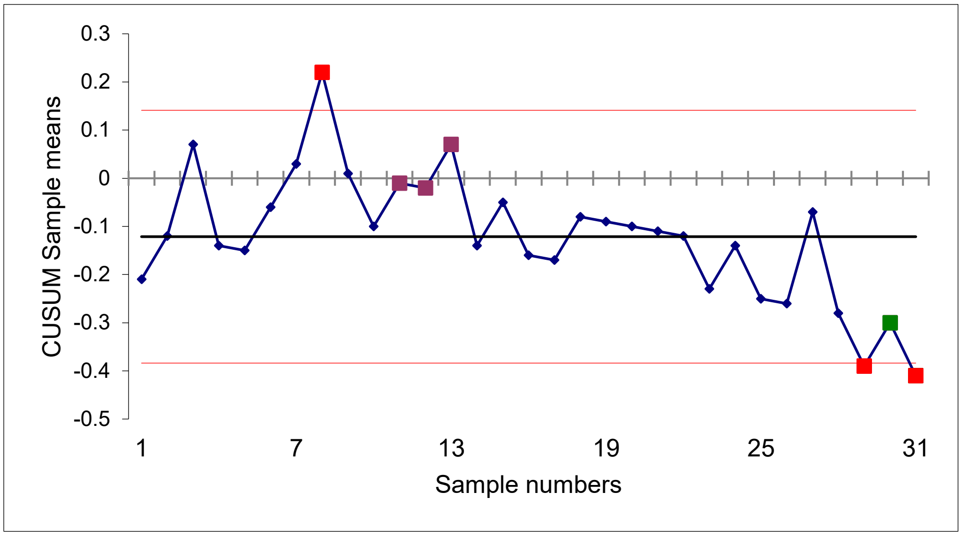

Fig. 2:CUSUM Chart showing in and out of control variables in August

As shown in Table 2, the deviations of the daily sample means from the target value ranged from −0.21 kg to +0.19 kg, indicating that individual daily measurements differed only slightly from the expected process average. Consequently, the cumulative sums fluctuated around the centre line throughout the monitoring period, reflecting the generally stable nature of the shortening dispensing process. However, the cumulative accumulation of these small deviations resulted in noticeable shifts at specific sampling points. Although the Shewhart Mean ($\bar{X}$) chart presented in Fig. 1 indicated that all 31 observations were within the prescribed control limits, the CUSUM chart in Fig. 2 revealed evidence of small but statistically significant process shifts. Specifically, three observations were identified as out-of-control signals, with one point exceeding the upper control limit and two points falling below the lower control limit. These signals correspond approximately to Day 8, Day 29, and Day 31, suggesting the occurrence of assignable causes that were not sufficiently large to be detected by the conventional $\bar{X}$ chart.

The upward excursion observed around Day 8 indicates a slight but persistent increase in the amount of shortening used relative to the target mean, whereas the downward signals recorded towards the end of the month (Days 29 and 31) indicate a gradual reduction in shortening usage. Although these deviations are relatively small, their cumulative effect is sufficient to trigger the CUSUM decision limits, demonstrating that the process experienced minor systematic shifts rather than purely random fluctuations. Such shifts may arise from gradual equipment calibration drift, minor inconsistencies in weighing operations, operator-related differences, or changes in raw material handling procedures. Consequently, these specific production days should be investigated to identify and eliminate potential assignable causes before they develop into more substantial process deviations.

The comparison between Figs 1 and 2 clearly demonstrates the complementary strengths of the two statistical process control techniques. While the Shewhart $\bar{X}$ chart confirmed that the overall process remained statistically stable with no individual observation exceeding the control limits, the CUSUM chart successfully detected subtle and persistent changes that accumulated over time. This finding highlights the superior sensitivity of the CUSUM technique for identifying small process shifts, making it particularly valuable in food manufacturing operations where even minor ingredient variations can influence product quality, texture, and consistency.

The present findings agree with previous studies that have reported the greater sensitivity of CUSUM charts compared with Shewhart charts in detecting gradual process changes. Nightingale et al. (2012) demonstrated that CUSUM monitoring provides earlier detection of small process deviations than conventional Shewhart charts, thereby allowing timely corrective actions before product quality is adversely affected. Similarly, Aslam et al. (2021) and Hussain et al. (2025) reported that CUSUM-based monitoring techniques outperform traditional control charts in identifying persistent but relatively small variations in industrial production processes. The current results therefore confirm that, although the shortening dispensing process was generally stable during August, the CUSUM chart provided an additional level of surveillance by identifying emerging process shifts that would otherwise have remained undetected.

Overall, the combined interpretation of Table 2, Fig. 1, and Fig. 2 indicates that the shortening dispensing process operated under satisfactory statistical control during most of the production period. Nevertheless, the CUSUM chart revealed three early warning signals that warrant further investigation. These findings underscore the importance of employing both Shewhart $\bar{X}$ and CUSUM control charts in quality control programmes, as their combined application provides a more comprehensive assessment of process stability by detecting both large, abrupt changes and small, gradual shifts in production performance.

3.3 Analysis of the Weight of Shortening Used in Bread Production (kg) during the Month of September Using the Shewhart Mean ($\bar{X}$) Control Chart

Table 3 summarizes the daily measurements of shortening (kg) used in bread production during the month of September, while Fig. 3 presents the corresponding Shewhart Mean ($\bar{X}$) control chart. The daily sample means ranged from 11.7 to 12.2 kg, indicating that the quantity of shortening used remained fairly consistent throughout the month. Similarly, the sample ranges varied from 0.1 to 0.8 kg, reflecting relatively low within-sample variability and suggesting that the production process maintained a high degree of consistency.

As illustrated in Fig. 3, all the sample means are located within the established Upper Control Limit (UCL) and Lower Control Limit (LCL). None of the thirty (30) observations exceeded the control limits, and the plotted points exhibit a random distribution around the centre line. There are no prolonged upward or downward trends, cycles, or runs of consecutive points on one side of the centre line that would indicate the presence of special-cause variation. This pattern demonstrates that the shortening dispensing process remained statistically stable throughout the month of September.

The results presented in Table 3 further support the graphical evidence from Fig. 3. Although slight fluctuations in the daily sample means were observed, these variations were relatively small and are characteristic of common-cause variation normally associated with routine manufacturing processes. For example, the lowest sample mean of 11.7 kg recorded on Day 30 and the highest sample means of 12.2 kg recorded on Days 4, 12, 17, 22, and 29 remained well within the prescribed control limits. Likewise, the largest sample ranges of 0.8 kg recorded on Days 9, 15, and 23 did not result in any corresponding shift beyond the control boundaries, indicating that these variations were insufficient to compromise overall process stability.

Table 3: Analysis on weight of shortening fused in production of bread in (kg) in the month of September

S/No.	Daily Readings					
Days	X1	X2	X3	X4	Mean ($\overline{X}$)	Range
1	12.0	12.4	11.9	12.0	12.1	0.5
2	11.7	12.2	11.8	11.6	11.8	0.6
3	12.3	12.0	11.8	12.3	12.1	0.5
4	12.1	12.4	12.3	12.0	12.2	0.4
5	11.6	12.1	11.9	12.0	11.9	0.5
6	11.9	12.3	11.8	12.1	12.0	0.5
7	11.8	11.6	12.0	11.9	11.8	0.4
8	12.3	12.0	11.8	11.6	11.9	0.7
9	12.3	11.6	12.0	12.4	12.1	0.8
10	11.8	11.9	12.0	12.2	12.0	0.6
11	12.1	11.7	12.3	11.9	12.0	0.6
12	11.8	12.2	12.0	12.1	12.2	0.4
13	12.2	12.3	12.0	11.7	12.0	0.6
14	12.1	11.8	11.9	11.6	11.8	0.3
15	12.41	11.6	12.4	11.7	12.0	0.8
16	11.8	11.7	11.8	12.1	11.9	0.4
17	12.0	12.3	12.3	12.1	12.2	0.2
18	11.9	11.9	11.9	12.2	12.0	0.3
19	12.3	12.0	12.0	11.8	12.1	0.6
20	11.9	11.8	11.8	11.8	11.8	0.1
21	11.8	12.2	12.2	12.4	12.1	0.6
22	12.2	12.0	12.3	12.2	12.2	0.3
23	12.4	12.5	11.2	11.9	12.1	0.8
24	11.8	12.3	12.1	11.8	12.0	0.5
25	11.6	12.0	12.9	12.3	12.0	0.7
26	12.1	12.0	11.9	11.8	12.0	0.3
27	11.8	12.0	12.2	11.7	11.9	0.5
28	12.0	11.7	11.8	12.1	11.9	0.4
29	12.2	12.4	11.9	12.3	12.2	0.5
30	11.8	11.7	11.8	12.1	11.7	0.4

The stable pattern observed in the Shewhart $\bar{X}$ chart suggests that the bakery maintained effective control over the quantity of shortening incorporated into bread production during September. Such consistency is essential in commercial baking because shortening significantly influences dough rheology, loaf volume, crumb structure, texture, and product shelf life. The ability to maintain ingredient usage within acceptable statistical limits contributes to improved product uniformity, reduced process variability, and enhanced customer satisfaction.A comparison of the September results with those obtained for August (Table 1 and Fig. 1) reveals a similar level of process stability. In both months, the sample means remained within the control limits, indicating that the production process was consistently capable of maintaining the desired quantity of shortening. However, September exhibited slightly more frequent fluctuations between 11.8 and 12.2 kg, although these changes remained entirely attributable to common-cause variation and therefore do not indicate deterioration in process performance.

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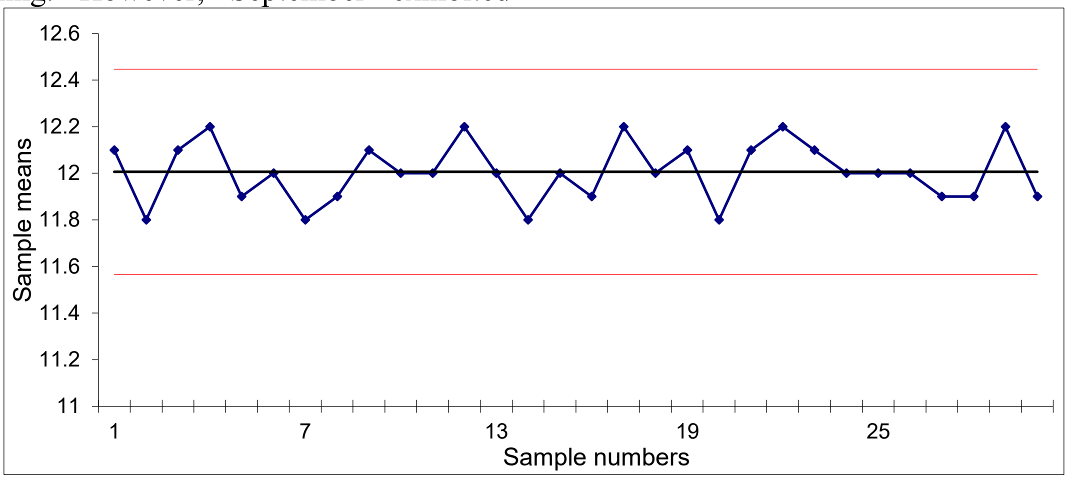

Fig. 3: Shewart Chart showing in and out of control variables in September

hese findings are consistent with the fundamental principles of Statistical Process Control (SPC), which state that a process is considered to be under statistical control when all observations remain within the control limits and display only random variation (Aslam et al., 2021). Similar observations have been reported by Nightingale et al. (2012), who demonstrated that Shewhart control charts effectively monitor production processes by distinguishing between natural process variation and assignable causes. Likewise, Hussain et al. (2025) emphasized that Shewhart charts provide an efficient first-level monitoring tool for identifying major process shifts while maintaining operational stability.

3.4 Analysis of the Weight of Shortening Used in Bread Production (kg) during the Month of September Using the Cumulative Sum (CUSUM) Control Chart

Table 4 presents the cumulative sum (CUSUM) statistics for the daily mean weights of shortening used in bread production during the month of September, while Fig. 4 illustrates the corresponding CUSUM control chart. The CUSUM values were obtained by cumulatively summing the deviations of the daily sample means from the target process mean of 12.01 kg. This method is more sensitive than the Shewhart Mean ($\bar{X}$) chart because it accumulates small deviations over time, thereby enabling the detection of gradual shifts in process performance.

As shown in Table 4, the deviations of the daily sample means from the target value ranged from −0.19 kg to +0.21 kg, indicating relatively small fluctuations around the expected process mean. Although these individual deviations appear insignificant, the cumulative values reveal gradual changes in the process behaviour. The cumulative sums initially fluctuated around the centre line before showing a steady upward trend from the latter part of the month, suggesting the presence of persistent positive deviations.

Unlike the Shewhart Mean ($\bar{X}$) chart (Fig. 3), which indicated that all observations remained within the control limits, the CUSUM chart

(Fig. 4) detected eight out-of-control observations among the thirty production days. Of these, six points exceeded the upper control limit, while two points fell below the lower control limit, indicating that the process experienced small but persistent shifts that were not evident in the conventional control chart.

The out-of-control signals occurred approximately on Days 2, 8, 23, 24, 25, 26, 29, and 30. The isolated signals observed on Days 2 and 8 suggest temporary deviations from the target process mean, whereas the consecutive signals recorded between Days 23 and 26 indicate a sustained upward shift in the process. Such sequential out-of-control observations are more likely to be associated with assignable causes than with random process variation. Possible sources include gradual equipment calibration errors, inconsistencies in ingredient weighing, operator-related differences, or changes in raw material characteristics. Consequently, these production days should be carefully investigated to identify and eliminate the underlying causes of variation.

A comparison of Figs 3 and 4 clearly demonstrates the greater sensitivity of the CUSUM chart. While the Shewhart $\bar{X}$ chart suggested that the shortening dispensing process was operating under statistical control, the CUSUM chart identified subtle cumulative deviations that accumulated over time. This confirms that the CUSUM technique is better suited for detecting small and persistent process shifts that may eventually affect product quality if left uncorrected.

Table 4: Analysis on weight of shortening fused in production of bread in (kg) in the month of September

Sample, i	X_i	$(X_i – 12.01)$	$C_i = (X_i – 12.01) + C_{i - 1}$
1	12.1	0.11	0.11
2	11.8	-0.19	-0.08
3	12.1	0.11	0.03
4	12.2	0.21	0.15
5	11.9	-0.09	0.24
6	12.0	0.01	0.16
7	11.8	-0.19	-0.03
8	11.9	-0.09	-0.12
9	12.1	0.11	-0.01
10	12.0	0.01	0
11	12.0	0.01	0.01
12	12.2	0.21	0.22
13	12.0	0.01	0.23
14	11.8	-0.19	0.04
15	12.0	0.01	0.05
16	11.9	-0.09	-0.04
17	12.2	0.21	0.17
18	12.0	0.01	0.18
19	12.1	0.11	0.19
20	11.8	-0.19	0.1
21	12.1	0.11	0.21
22	12.2	0.21	0.42

23	12.1	0.11	0.53
24	12.0	0.01	0.54
25	12.0	0.01	0.55
26	12.0	0.01	0.56
27	11.9	-0.09	0.47
28	11.9	-0.09	0.38
29	12.2	0.21	0.59
30	11.9	-0.09	0.5

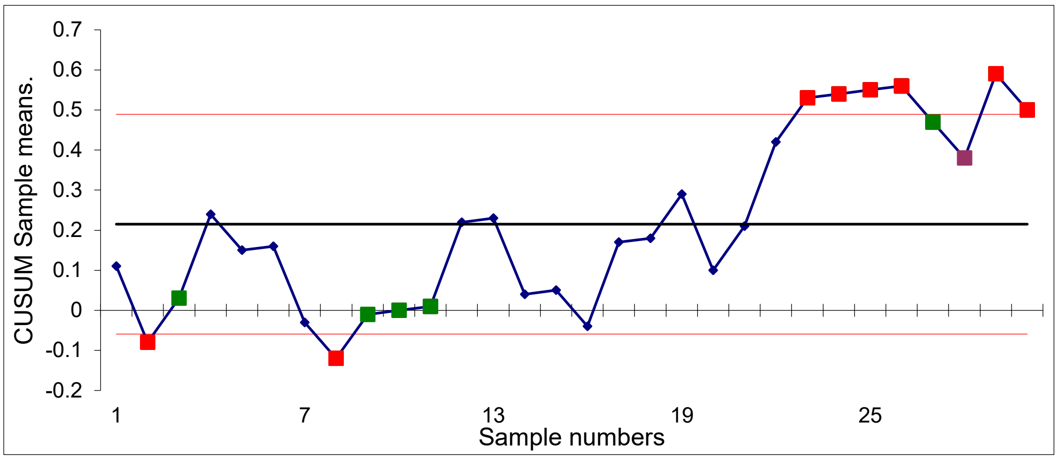

Fig. 4: CUSUM Chart showing in and out of control variables in September

The findings are consistent with those of Birdsall and Watts (2025), who reported that CUSUM charts provide earlier detection of process deterioration than conventional Shewhart charts. Similarly, Aslam et al. (2021) and Hussain et al. (2025) demonstrated that CUSUM monitoring is highly effective in identifying gradual process changes that remain undetected by traditional SPC methods. The results also agree with Nightingale et al. (2012), who emphasized that cumulative monitoring techniques offer improved sensitivity for quality control applications.

Overall, the results presented in Table 4 and Fig. 4 indicate that although the shortening dispensing process appeared to be stable according to the Shewhart Mean ($\bar{X}$) chart, the CUSUM chart revealed several early warning signals of process instability. The occurrence of eight out-of-control observations, particularly the consecutive signals between Days 23 and 26, suggests that the process was approaching a state of instability and required timely investigation and corrective action. These findings highlight the importance of incorporating CUSUM charts into routine quality monitoring to complement Shewhart control charts and enhance the early detection of subtle process variations in bread production.

3. 5 Analysis on weight of shortening fused in production of bread in (kg) in the month of October using the ean charts ($\bar{X}$ -chart)

Table 4 presents the cumulative sum (CUSUM) statistics for the daily mean weights of shortening used in bread production during the month of September, while Fig. 4 shows the corresponding CUSUM control chart. The CUSUM values were computed by cumulatively summing the deviations of the daily sample means from the target process mean of 12.01 kg. This technique is particularly effective in detecting small and persistent shifts in process performance that may not be identified by the conventional Shewhart X̄ chart.

As shown in Table 4, the deviations of the daily sample means from the target value ranged from −0.19 kg to +0.21 kg, indicating relatively small day-to-day departures from the expected process mean. Although these deviations were individually insignificant, their cumulative effect produced noticeable changes in the CUSUM statistics over the monitoring period. The cumulative sums initially fluctuated around the centre line during the first three weeks of production before exhibiting a gradual upward trend towards the end of the month.

Table 5: Analysis on weight of shortening fused in production of bread in (kg) in October

S/No.	**Daily Readings**					
Days	X1	X2	X3	X4	Mean ($\overline{X}$)	Range
1	12.1	11.6	12.4	11.7	12.0	0.8
2	11.7	11.8	11.9	11.6	11.8	0.3
3	11.8	12.3	12.2	11.7	12.0	0.6
4	12.4	12.2	12.0	12.1	12.2	0.4
5	12.1	11.7	12.3	11.9	12.0	0.6
6	11.8	11.9	12.0	12.2	12.0	0.4
7	12.3	12.0	11.8	11.6	11.9	0.7
8	11.8	11.6	12.0	11.9	11.8	0.4
9	11.9	12.3	11.8	12.1	12.0	0.5
10	11.6	12.1	11.9	12.0	11.9	0.5
11	12.1	12.4	12.3	12.0	12.2	0.4
12	12.3	12.0	11.8	11.6	12.1	0.5
13	11.7	12.2	11.8	11.6	11.8	0.6
14	12.0	12.4	11.9	12.0	12.1	0.5
15	11.8	11.7	11.8	12.1	11.9	0.4
16	12.2	12.4	11.9	12.3	12.2	0.5
17	12.0	11.7	11.8	12.1	11.9	0.4
18	11.8	12.0	12.1	11.7	11.9	0.4
19	12.1	12.0	11.9	11.8	12.0	0.3
20	11.6	12.0	12.2	12.0	12.0	0.6
21	11.8	12.3	12.1	11.8	12.0	0.5
22	12.4	12.5	11.7	11.9	12.1	0.8
23	12.2	12.0	12.3	12.2	12.2	0.3
24	12.0	11.8	12.2	12.4	12.1	0.6
25	11.8	11.9	11.8	11.8	11.8	0.1
26	12.4	12.2	12.0	11.8	12.1	0.6
27	12.1	11.9	11.9	12.2	12.0	0.3
28	12.2	12.3	12.3	12.1	12.2	0.2
29	11.8	11.7	11.8	12.1	11.9	0.4
30	-	-	-	-	-	-

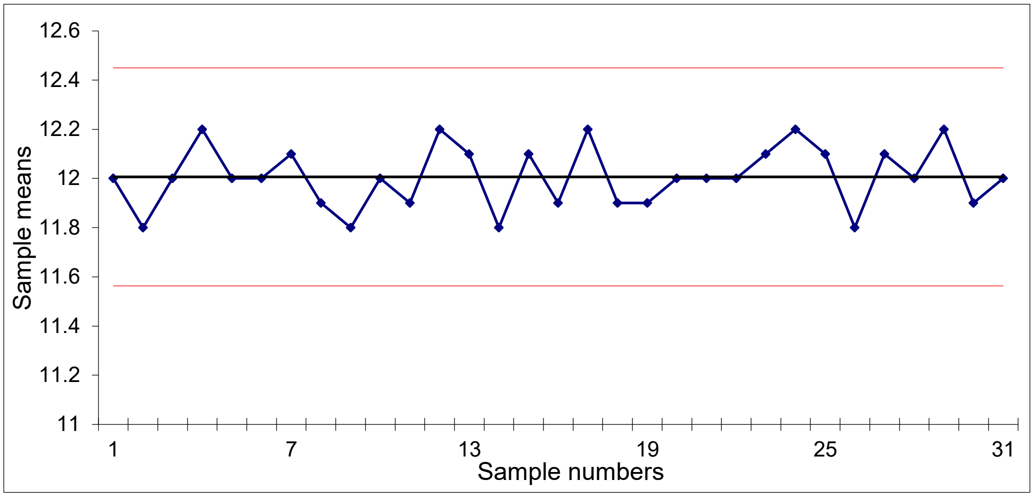

Fig. 5: Shewart Chart showing in and out of control variables in October

The Shewhart Mean ($\bar{X}$) chart (Fig. 3) indicated that all thirty (30) observations remained within the prescribed control limits, suggesting that the shortening dispensing process was statistically stable throughout September. However, the more sensitive CUSUM chart (Fig. 4) detected eight out-of-control signals among the thirty observations. Specifically, six observations exceeded the upper control limit, while two observations fell below the lower control limit. The out-of-control signals occurred approximately on Days 2, 8, 23, 24, 25, 26, 29, and 30, indicating the presence of small but persistent process shifts that were not sufficiently large to be identified by the Shewhart chart.

The sequence of out-of-control observations between Days 23 and 26 is particularly noteworthy. Rather than representing isolated random events, these consecutive signals suggest the possibility of a sustained upward shift in the quantity of shortening used during this period. Such sequential violations of the CUSUM decision limits often indicate the influence of an assignable cause, such as gradual equipment calibration drift, changes in operator practices, inaccuracies in ingredient weighing, or variations in raw material characteristics. Consequently, these production days warrant detailed investigation to determine whether the observed deviations resulted from identifiable operational factors or merely reflected prolonged random variation.

The comparison between Figs 3 and 4 further highlights the complementary roles of the Shewhart and CUSUM control charts in statistical process monitoring. While the Shewhart chart effectively confirmed the absence of large process disturbances, the CUSUM chart revealed subtle but persistent shifts in the process mean that accumulated over time. This demonstrates the superior sensitivity of the CUSUM approach for early detection of gradual process deterioration, thereby enabling corrective actions before substantial deviations occur or product quality is compromised.

A comparison with the August results (Table 2 and Fig. 2) also reveals that the shortening dispensing process exhibited greater cumulative variability during September. Whereas only three out-of-control signals were detected by the August CUSUM chart, eight signals were observed in September, suggesting that the process experienced more frequent small shifts during the latter month despite remaining within the Shewhart control limits. This finding indicates that, although the process was generally stable, there was a greater tendency for systematic deviations to accumulate during September, emphasizing the importance of continuous process monitoring.

The present findings agree with previous studies that have demonstrated the effectiveness of CUSUM charts in detecting small and sustained process changes. Birdsall and Watts (2025) observed that CUSUM charts are capable of identifying early process deterioration long before conventional Shewhart charts signal an out-of-control condition. Similarly, Aslam et al. (2021) reported that CUSUM monitoring provides greater sensitivity for identifying gradual process shifts under routine industrial operations, while Hussain et al. (2025) further showed that enhanced CUSUM procedures improve the detection of persistent deviations in quality control applications. These findings are also consistent with the work of Nightingale et al. (2012), who demonstrated that cumulative monitoring techniques provide earlier warning of process instability than traditional Shewhart control charts.

Overall, the combined evidence from Table 4, Fig. 3, and Fig. 4 indicates that the shortening dispensing process remained largely stable during September when evaluated using the Shewhart $\bar{X}$ chart. Nevertheless, the CUSUM chart identified several early warning signals, particularly the consecutive out-of-control observations between Days 23 and 26, which suggest the emergence of gradual process shifts. Although these signals do not imply complete loss of process control, they indicate that the process was approaching a state of instability and therefore required timely investigation and corrective action. The findings underscore the importance of integrating Shewhart $\bar{X}$ and CUSUM control charts in bakery quality management systems to ensure both immediate detection of major process changes and early identification of subtle, cumulative variations that may adversely affect product consistency.

3.6: Analysis on weight of shortening fused in production of bread in (kg) in the month of October using the CUSUM charts

Table 6 presents the cumulative sum (CUSUM) statistics for the daily mean weights of shortening used in bread production during the month of October, while Fig. 6 illustrates the corresponding CUSUM control chart. The CUSUM values were calculated by cumulatively summing the deviations of the daily sample means from the target process mean of 12.01 kg. This approach provides a more sensitive measure of process performance than the Shewhart $\bar{X}$ chart because it accumulates small deviations over time, thereby enabling the early detection of gradual process shifts.

As shown in Table 6, the daily deviations from the target mean varied between −0.19 kg and +0.30 kg, indicating relatively small day-to-day fluctuations in the quantity of shortening used during bread production. Although these individual deviations appear minor, their cumulative effect resulted in a progressive upward movement of the CUSUM statistic, particularly during the latter part of the month. This pattern suggests that small systematic deviations accumulated over time rather than occurring as isolated random events.

The Shewhart Mean ($\bar{X}$) chart presented in Fig. 5 indicated that all thirty-one (31) sample means remained within the established control limits, implying that the process was statistically stable when evaluated using conventional control chart techniques. However, the CUSUM chart (Fig. 6) provided a different perspective by detecting thirteen (13) out-of-control signals among the thirty-one observations. Of these, seven observations exceeded the upper control limit, while six observations fell below the lower control limit, indicating that the process experienced substantial cumulative variation despite the absence of individual points outside the Shewhart control limits.

Table 6: Analysis on weight of shortening fused in production of bread in (kg) in the month of October

Sample, i	X_i	$(X_i – 12.01)$	$C_i = (X_i – 12.01) + C_i - 1$
1	12.1	0.01	0.01
2	11.8	-0.19	-0.18
3	12.0	0.01	-0.17
4	12.2	0.3	0.13
5	12.0	0.01	0.14
6	12.0	0.01	0.15
7	12.1	0.11	0.26
8	11.9	-0.09	0.17
9	11.8	-0.19	-0.02
10	12.0	0.01	-0.01
11	11.9	-0.09	-0.1
12	12.2	0.3	0.2
13	12.1	0.11	0.31
14	11.8	-0.19	0.12
15	12.1	0.11	0.23
16	11.9	-0.09	0.14
17	12.2	0.3	0.44
18	11.9	-0.09	0.35
19	11.9	-0.01	0.26
20	12.0	0.01	0.27
21	12.0	0.01	0.28
22	12.0	0.01	0.29
23	12.1	0.11	0.4
24	12.2	0.3	0.7
25	11.8	-0.19	0.81
26	12.1	0.11	0.73
27	12.0	0.01	0.74
28	12.2	0.3	1.04
29	11.9	-0.09	0.95
30	12.0	0.01	0.96
31	12.1	0.11	0.4

The out-of-control signals occurred approximately on Days 1, 2, 3, 7, 8, 9, 24, 25, 27, 28, 29, 30, and 31, suggesting that the shortening dispensing process experienced repeated and persistent deviations from the target value. Particularly noteworthy are the consecutive out-of-control observations recorded during Days 24–25 and Days 27–31, which indicate a sustained shift in the process mean rather than isolated random fluctuations. Such prolonged sequences are generally associated with assignable causes, including gradual equipment miscalibration, inconsistencies in weighing procedures, operator-related differences, changes in production practices, or variations in the physical properties of the shortening. These periods therefore require detailed investigation to determine the underlying causes and to implement appropriate corrective actions before product quality is adversely affected.

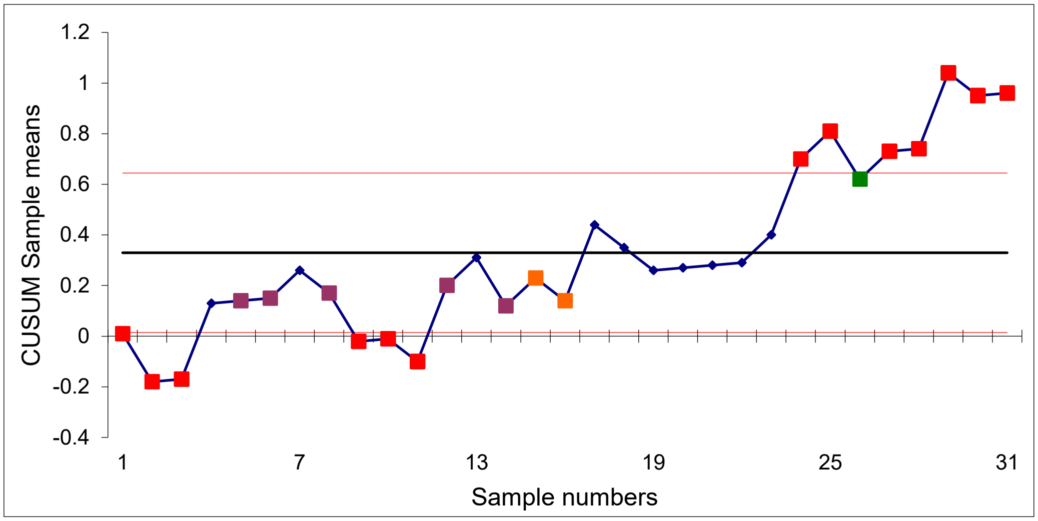

Fig. 6: CUSUM Chart showing in and out of control variables in October

The comparison between Figs 5 and 6 clearly demonstrates the increased sensitivity of the CUSUM chart relative to the Shewhart $\bar{X}$ chart. Whereas the Shewhart chart detected no evidence of process instability because all observations remained within the control limits, the CUSUM chart successfully identified numerous small but persistent shifts that accumulated over time. This finding confirms that relying solely on Shewhart charts may provide a false impression of complete process stability when gradual systematic changes are occurring. The CUSUM technique therefore serves as an effective complementary tool for continuous process monitoring, particularly in food manufacturing where small ingredient deviations can significantly influence product quality and consistency.

A comparison of the October results with those obtained in August (Table 2 and Fig. 2) and September (Table 4 and Fig. 4) further illustrates the progressive increase in cumulative process variation over the three-month study period. The August CUSUM chart identified three out-of-control observations, while eight signals were detected in September. In contrast, the October CUSUM chart revealed thirteen out-of-control observations, representing the highest level of cumulative process instability observed during the study. This trend suggests that the shortening dispensing process became increasingly susceptible to persistent deviations over time, despite remaining within the Shewhart control limits. Consequently, October exhibited the greatest likelihood of assignable causes influencing the production process.

The present findings are consistent with previous studies that have demonstrated the superior capability of CUSUM charts in identifying gradual process deterioration. Birdsall and Watts (2025) reported that cumulative monitoring techniques provide early warning signals for sustained process shifts before they become detectable using conventional Shewhart control charts. Similarly, Aslam et al. (2021) showed that CUSUM control charts are highly effective in detecting persistent small changes in industrial production systems, while Hussain et al. (2025)

further demonstrated that enhanced CUSUM procedures improve sensitivity for monitoring process quality. The observations also agree with Nightingale et al. (2012), who reported that cumulative statistical monitoring techniques outperform traditional Shewhart charts in identifying early signs of process instability.

Overall, the results presented in Table 6, together with Figs 5 and 6, indicate that although the shortening dispensing process appeared to be statistically stable according to the Shewhart $\bar{X}$ chart, the CUSUM chart revealed considerable cumulative variation during October. The detection of thirteen out-of-control signals, particularly the consecutive violations observed during Days 24–25 and Days 27–31, indicates that the process was approaching a state of statistical instability and required immediate investigation. These findings emphasize the importance of incorporating both Shewhart $\bar{X}$ and CUSUM control charts into bakery quality management systems, as their combined application provides comprehensive monitoring of both abrupt process changes and subtle cumulative shifts, thereby ensuring more effective process control and consistent product quality.

3.7 Analysis of the Weight of Shortening Used in Bread Production (kg) during the Month of November Using the Shewhart Mean ($\bar{X}$) Control Chart

Table 7 presents the daily measurements of shortening (kg) used in bread production during the month of November, while Fig. 7 illustrates the corresponding Shewhart Mean ($\bar{X}$) control chart. The daily sample means ranged from 11.7 to 12.2 kg, indicating that the quantity of shortening used remained relatively consistent throughout the production period. The sample ranges varied from 0.1 to 0.8 kg, reflecting low within-sample variability and demonstrating good consistency in the ingredient measurement process.

As shown in Fig. 7, all thirty (30) sample means are located within the established Upper Control Limit (UCL) and Lower Control Limit (LCL), with no observations exceeding the control boundaries. Furthermore, the plotted points fluctuate randomly around the centre line without exhibiting systematic trends, prolonged runs, cyclic patterns, or sudden shifts that would indicate the presence of assignable causes. This random distribution confirms that the shortening dispensing process remained statistically stable throughout the month of November.

The statistical stability observed in Fig. 7 is supported by the descriptive results presented in Table 7. Most of the daily sample means were concentrated around the target value of approximately 12.0 kg, indicating that the bakery maintained close adherence to the required shortening formulation. The lowest sample mean of 11.7 kg, recorded on Day 6, and the highest sample means of 12.2 kg, recorded on Days 5, 8, 15, 25, and 29, remained well within the control limits and therefore represent normal process variation rather than evidence of process instability. Likewise, although relatively larger sample ranges of 0.8 kg were observed on Days 2 and 29, these did not correspond to any abnormal movement of the process mean, suggesting that the observed variability was attributable to common-cause variation inherent in routine production activities.

The findings indicate that the bakery maintained effective control over the quantity of shortening incorporated into bread production throughout November. Consistent ingredient measurement is essential in commercial bread production because shortening directly influences dough handling characteristics, crumb softness, loaf volume, texture, and product shelf life. Maintaining the process within statistical control therefore contributes significantly to ensuring product uniformity, minimizing production variability, and improving overall manufacturing efficiency.

A comparison of the November results with those obtained in the preceding months (Tables 1, 3, and 5; Figs 1, 3, and 5) reveals a similar pattern of process stability. Like the months of August, September, and October, all observations in November remained within the Shewhart control limits, confirming that the shortening dispensing process consistently operated under statistical control. However, November exhibited slightly lower variability than October, with fewer pronounced fluctuations in the daily sample means and a more balanced distribution of observations around the centre line. This suggests a slight improvement in process consistency during the month.

Table 7: Analysis on weight of shortening fused in production of bread in (kg) in the month of November

S/No.	Daily Readings					
Day	X1	X2	X3	X4	Mean ($\overline{X}$)	Range
1	11.8	11.7	12.1	11.8	11.9	0.4
2	12.1	11.6	11.7	12.4	12.0	0.8
3	12.0	12.4	12.0	11.9	12.1	0.5
4	11.8	11.7	12.0	11.8	11.8	0.3
5	12.2	12.3	12.1	12.3	12.2	0.2
6	11.7	11.8	11.6	11.7	11.7	0.3
7	11.7	12.1	11.6	11.8	11.8	0.5
8	12.2	12.4	12.3	11.9	12.2	0.5
9	12.1	11.9	12.2	11.9	12.0	0.3
10	11.8	12.3	11.7	12.0	12.0	0.6
11	12.3	12.0	12.3	11.8	12.1	0.5
12	12.0	11.7	12.1	11.8	121.9	0.4
13	12.4	12.2	11.8	12.0	12.1	0.6
14	12.4	12.2	11.9	12.0	12.1	0.5
15	12.1	12.4	12.0	12.3	12.2	0.4
16	11.8	12.0	11.9	12.1	12.0	0.3
17	11.8	11.9	11.8	11.8	11.8	0.1
18	12.1	11.7	121.9	12.3	12.0	0.4
19	11.6	12.1	12.0	11.9	11.9	0.5
20	12.1	12.0	11.8	11.9	12.0	0.3
21	12.0	11.8	12.4	12.2	12.1	0.6
22	11.8	11.9	12.2	12.0	12.0	0.4
23	11.9	12.3	12.1	11.8	12.0	0.5
24	11.6	12.0	12.3	12.2	12.0	0.7
25	12.2	12.0	12.2	12.3	12.2	0.3
26	12.3	11.6	12.2	12.0	12.0	0.7
27	11.8	11.6	11.9	12.0	11.8	0.4
28	11.8	12.3	11.8	12.1	12.0	0.5
29	12.4	12.5	11.9	11.7	12.2	0.8
30	12.3	12.0	11.6	11.8	11.9	0.7

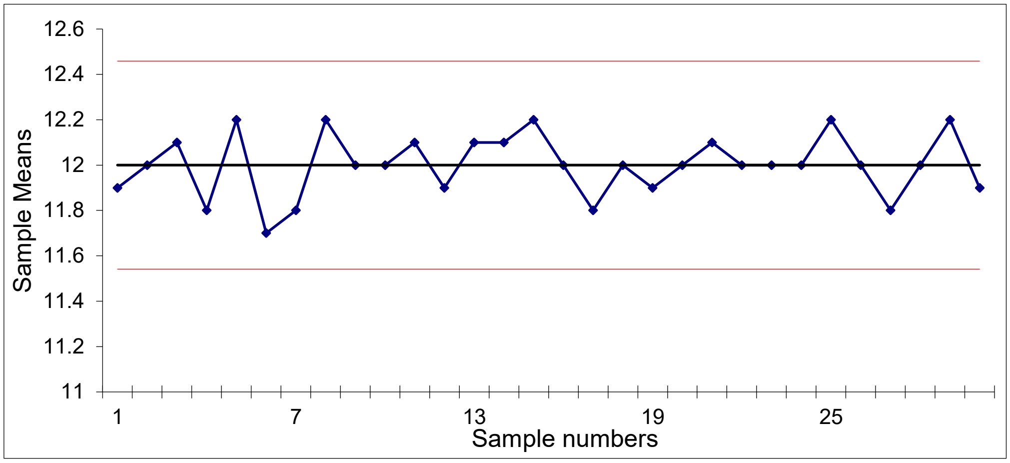

Fig. 7: Shewart Chart showing in and out of control variables in November

These findings are consistent with the principles of Statistical Process Control (SPC), which state that a process is considered to be in statistical control when observations remain within the prescribed control limits and display only random variation arising from common causes (Aslam et al., 2021). Similar conclusions were reported by Nightingale et al. (2012), who demonstrated that Shewhart control charts provide an effective means of monitoring production processes by distinguishing between natural process variability and assignable causes. Likewise, Hussain et al. (2025) emphasized that Shewhart $\bar{X}$ charts are suitable for identifying major process shifts while confirming the overall stability of manufacturing operations.

Overall, the results presented in Table 7 and Fig. 7 indicate that the quantity of shortening used in bread production during November remained under statistical control. The absence of out-of-control observations demonstrates that the production process consistently maintained the required shortening levels throughout the month. These findings provide evidence of effective process management and reinforce the suitability of the Shewhart $\bar{X}$ chart as a reliable tool for routine monitoring of ingredient usage in bread manufacturing.

3.8 Analysis of the Weight of Shortening Used in Bread Production (kg) during the Month of November Using the Cumulative Sum (CUSUM) Control Chart

Table 8 presents the cumulative sum (CUSUM) statistics for the daily mean weights of shortening used in bread production during the month of November, while Fig. 8 illustrates the corresponding CUSUM control chart. The CUSUM values were obtained by cumulatively summing the deviations of the daily sample means from the target process mean of 12.01 kg. Unlike the Shewhart Mean ($\bar{X}$) chart, which is primarily designed to identify large and sudden process shifts, the CUSUM chart accumulates small deviations over time, making it more effective in detecting gradual changes in process performance.

As presented in Table 8, the daily deviations from the target mean ranged between −0.30 kg and +0.20 kg, indicating only minor fluctuations in the quantity of shortening used throughout the production period. The cumulative sums oscillated closely around the centre line, with values ranging from −0.50 to +0.10, suggesting that the process remained relatively stable and that no sustained upward or downward drift occurred during most of the month.

The Shewhart Mean ($\bar{X}$) chart shown in Fig. 7 indicated that all thirty (30) sample means were within the established Upper Control Limit (UCL) and Lower Control Limit (LCL), confirming that the shortening dispensing process was statistically stable when evaluated using conventional control chart techniques. However, when the same observations were analysed using the more sensitive CUSUM chart (Fig. 8), only one out-of-control signal was detected among the thirty observations. This single signal occurred below the lower control limit, corresponding approximately to Day 7, indicating a temporary downward shift in the amount of shortening used during that production day.

The isolated out-of-control signal suggests that the process experienced a brief deviation from the target mean rather than a sustained deterioration in performance. Such an occurrence may have resulted from a temporary assignable cause, including minor weighing inaccuracies, short-term equipment adjustment, operator variability, or a brief interruption in the ingredient dispensing process. Because the process immediately returned to values within the acceptable control region after this observation, the deviation is unlikely to represent a persistent process problem. Nevertheless, the production activities on Day 7 should be reviewed to determine whether the observed variation resulted from an identifiable operational factor or occurred purely by chance.

The comparison between Figs 7 and 8 further demonstrates the complementary roles of the Shewhart and CUSUM control charts. While the Shewhart chart confirmed that the production process remained within statistical control throughout November, the CUSUM chart provided an additional level of sensitivity by identifying a subtle process shift that was not sufficiently large to trigger the Shewhart control limits. This observation reinforces the value of CUSUM charts as early-warning tools capable of detecting minor process changes before they develop into significant quality problems.

A comparison of the CUSUM results across the four months of the study reveals a clear improvement in process stability during November. The August analysis (Table 2 and Fig. 2) identified three out-of-control observations, while September (Table 4 and Fig. 4) recorded eight out-of-control signals, and October (Table 6 and Fig. 6) exhibited the highest level of cumulative variation with thirteen out-of-control observations. In contrast, November (Table 8 and Fig. 8) recorded only one out-of-control observation, indicating that the shortening dispensing process achieved its highest level of statistical stability during the study period. This progressive improvement suggests that the production process became increasingly consistent, with fewer cumulative deviations from the target value.

The present findings agree with previous studies demonstrating the superior sensitivity of CUSUM charts for detecting small and persistent process shifts. Birdsall and Watts (2025) reported that CUSUM monitoring provides earlier detection of subtle process changes than conventional Shewhart charts, thereby enabling timely corrective action before significant quality deterioration occurs. Similarly, Aslam et al. (2021) and Hussain et al. (2025) showed that CUSUM-based monitoring techniques are particularly effective for identifying gradual deviations in manufacturing processes, while Nightingale et al. (2012) demonstrated that cumulative statistical monitoring substantially improves process surveillance compared with traditional Shewhart control charts.

Overall, the combined interpretation of Table 8, Fig. 7, and Fig. 8 indicates that the shortening dispensing process operated under excellent statistical control during November. The detection of only one out-of-control signal demonstrates that the production process consistently maintained the required quantity of shortening throughout the month, with only a minor and isolated deviation requiring investigation. Based on the CUSUM analyses for all four months, November exhibited the best process performance, followed by August, September, and October, which recorded the highest degree of cumulative process variation. These findings clearly demonstrate that, while the Shewhart $\bar{X}$ chart is effective for detecting major process disturbances, the CUSUM chart provides superior sensitivity for identifying small but important process shifts. Consequently, the combined application of both statistical process control techniques offers a more comprehensive and reliable approach for monitoring ingredient consistency and maintaining high-quality bread production

Table 8: Analysis on weight of shortening fused in production of bread in (kg) in the month of November

Sample, i	X_i	$(X_i - 12.01)$	$C_i = (X_i - 12.01) + C_i - 1$
1	11.9	-0.1	-0.1
2	12.0	0	-0.1
3	12.1	0.1	0
4	11.8	-0.2	-0.2
5	12.2	0.2	0
6	11.7	-0.3	-0.3
7	11.8	-0.2	-0.5
8	12.2	0.2	-0.3
9	12.0	0	-0.3
10	12.0	0	-0.3
11	12.1	0.1	-0.2
12	11.9	-0.1	-0.3
13	12.1	0.1	-0.2
14	12.1	0.1	-0.1
15	12.2	0.2	0.1
16	12.0	0	0
17	11.8	-0.2	-0.2
18	12.0	0	-0.2
19	11.9	-0.1	-0.3
20	12.0	0	-0.3
21	12.1	0.1	-0.2
22	12.0	0	-0.2
23	12.0	0	-0.2
24	12.0	0	-0.2
25	12.2	0.2	0
26	12.0	0	0
27	11.8	0.2	-0.2
28	12.0	0	-0.2
29	12.2	0.2	0
30	11.9	-0.1	-0.1

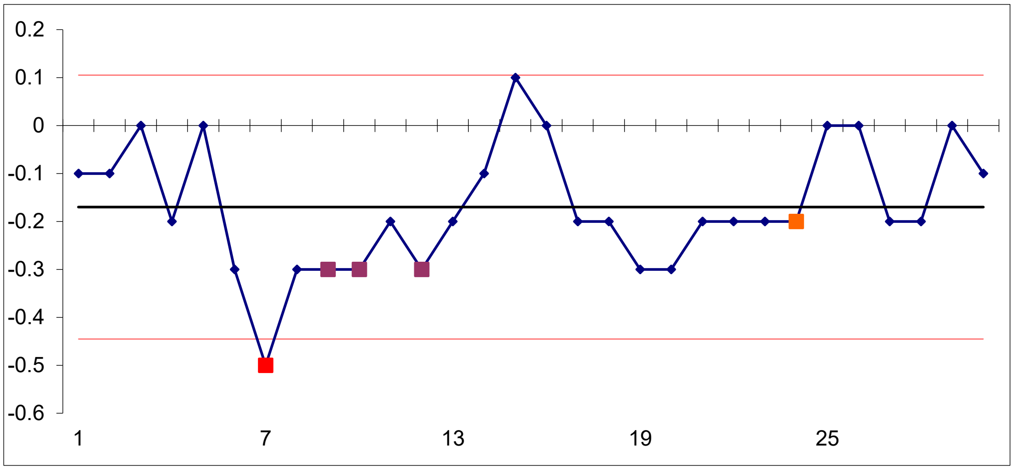

Fig. 8: CUSUM Chart showing in and out of control variables in November

4.0 Conclusion

Fig. 1 indicates that the process in August remained within control, as all shortening weights used in bread production fell within the control limits. However, analysis of the same data using the CUSUM chart (Fig. 2) revealed that three out of thirty-one points were out of control: one above the upper control limit and two below the lower control limit. While the process cannot be classified as entirely out of control, sources of variation on Days 8, 29, and 31 warrant further investigation. The CUSUM chart is thus able to detect small variations that the X-bar chart does not identify. Similarly, Fig. 3 shows that all points for September appear within control limits, but the CUSUM chart (Fig. 4) identified eight out-of-control points out of thirty: six above the upper control limit and two below the lower control limit. Although the process is not completely out of control, it is considered nearly out of control (Aimee Birdsall & Valerie Watts, 2025). The sources of variation on Days 2, 8, 23, 24, 25, 26, 29, and 30 should be examined, and the sequential out-of-control state from Days 23 to 26 should be assessed to determine whether the causes are assignable or random. In October (Fig. 5), all points again appear within control limits, but the CUSUM chart (Fig. 6) identified thirteen out-of-control points out of thirty-one: seven above the upper control limit and six below the lower control limit. This indicates the process was out of control, as confirmed by the number of out-of-control points (Aimee Birdsall & Valerie Watts, 2025). The sources of variation on Days 1, 2, 3, 7, 8, 9, 24, 25, 27, 28, 29, 30, and 31 should be investigated, and the sequential out-of-control states on Days 24-25 and 27-31 should be analyzed for assignable or random causes. Finally, Fig. 7 shows that all points for November were within control limits, but the CUSUM chart (Fig. 8) identified one out-of-control point out of thirty, which was below the lower control limit. The process is considered within control, as only one point was out of control (Aimee Birdsall & Valerie Watts, 2025), indicating that on Day 7, the weight of shortening used was inadequate. In summary, the production process most consistently met quality standards for shortening weights in November, followed by August, then September, and least effectively in October.

The CUSUM chart facilitates the detection of small variations that are not typically identified by the X-bar chart.

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Compliance with Ethical Standards

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Not applicable

Consent to Participate

Not applicable.

Consent for Publication

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Availability of Data and Materials

The datasets generated and analysed during this study are available from the corresponding author upon reasonable request.

Author Contributions

Anthony Ekpo designed the study, led the statistical methodology comparing the Shewhart and CUSUM control charts, and authored the manuscript. Kingsley Agah managed data collection on daily shortening weights from the bakery, contributed quality control management expertise, and supported data analysis and manuscript review.