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Application of The Interior Point and NSGA-II to Multi-Objective Linear Programming

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Abstract: *Multi-Objective Linear Programming (MOLP) provides a systematic framework for solving decision problems involving multiple, often conflicting, linear objectives. This study compares the performance of Arbel's Affine Scaling Interior Multi-Objective Linear Programming (ASIMOLP) algorithm and the Nondominated Sorting Genetic Algorithm II (NSGA-II) for solving MOLP problems. The comparison focuses on the quality of the nondominated solutions generated by the two approaches. ASIMOLP was implemented in MATLAB based on the affine scaling interior-point procedure, while an existing MATLAB implementation of NSGA-II was employed for the evolutionary optimization process. Both algorithms were tested on ten MOLP instances ranging from small to medium-sized problems, comprising benchmark and randomly generated instances with varying numbers of variables, constraints, and objectives. The computational results indicate that ASIMOLP generally produced nondominated solutions with better objective-function values than NSGA-II across most of the test problems. Although NSGA-II was capable of generating good approximations of the nondominated set, the solutions obtained were generally inferior to those produced by ASIMOLP for the instances considered. The findings demonstrate the effectiveness of the interior-point approach for obtaining high-quality nondominated solutions in MOLP and provide useful evidence for selecting solution methods according to the characteristics of the optimization problem. The study also highlights the potential of combining exact interior-point techniques with evolutionary approaches to improve the solution of larger and more complex multi-objective optimization problems..*

Keywords: *Multi-objective Linear Programming; Affine Scaling Interior Point Multi-objective Linear Programming Algorithm; Nondominated Sorting Genetic Algorithm (NSGA)-II; Nondominated points; Efficient solutions.*

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1.0 Introduction

Multi-Objective Linear Programming (MOLP) is an important class of mathematical optimization problems within Multiple Criteria Decision Making (MCDM), concerned with the simultaneous optimization of two or more, often conflicting, linear objective functions subject to a common set of constraints. Unlike single-objective linear programming, where an optimal solution can generally be identified according to a single criterion, MOLP recognizes that real-world decision problems frequently involve competing goals for which improvement in one objective may result in deterioration in another. Consequently, the solution process is concerned not only with identifying a single optimum but also with determining a set of efficient, Pareto-optimal, or nondominated solutions from which decision-makers can select a preferred alternative (Nyiam, 2019).

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The relevance of MOLP has expanded considerably because of its ability to represent complex decision environments involving competing economic, technical, environmental, and operational considerations. Multi-objective optimization has been applied to problems in power-system planning, energy management, building design, engineering design, resource allocation, transportation, and other fields in which several performance criteria must be considered simultaneously. For example, Ramesh et al. (2012) applied a modified Non-Dominated Sorting Genetic Algorithm II (MNSGA-II) to reactive power planning, simultaneously considering operating and reactive-power allocation costs, voltage-profile improvement, and voltage stability. Their results demonstrated the usefulness of evolutionary multi-objective optimization in obtaining diverse and high-quality Pareto solutions for complex engineering problems.

The growing complexity of contemporary optimization problems has also encouraged the application of multi-objective techniques beyond conventional mathematical programming. In building design, Zou et al. (2025) developed a multi-objective optimization framework that simultaneously addressed energy consumption, thermal comfort, and daylight performance. Their optimized designs achieved substantial improvements across the competing objectives, demonstrating the capacity of multi-objective approaches to support balanced decision-making. Similarly, Hashemi et al. (2025) proposed an integrated data-driven framework for multi-objective optimization of building forms at the early design stage, illustrating the increasing integration of computational modelling and optimization techniques in complex design problems. Alexakis et al. (2025), in a systematic review of genetic algorithm-based building retrofitting, further reported the increasing use of multi-objective genetic algorithms for balancing energy consumption, retrofit cost, thermal comfort, and other performance criteria. These developments demonstrate that the selection of an appropriate optimization algorithm can substantially influence the quality, diversity, and computational efficiency of the solutions obtained.

In power-system applications, multi-objective optimization has similarly become an important tool for addressing competing technical and economic requirements. Becerra et al. (2026) applied NSGA-II to the optimal siting and sizing of battery energy storage systems in the Mali transmission network, simultaneously considering economic returns and active-power losses while incorporating network constraints. The study demonstrated that evolutionary optimization can provide decision-makers with a Pareto front representing alternative trade-offs between competing system objectives. Katkar and Jadhav (2026) also demonstrated the usefulness of hybrid metaheuristic optimization for evaluating techno-economic and environmental objectives in renewable-integrated power-flow problems. Such applications reinforce the importance of reliable optimization algorithms capable of generating high-quality nondominated solutions in problems involving multiple and conflicting objectives.

The application of evolutionary algorithms to multi-objective optimization has received particular attention because conventional mathematical approaches may become computationally demanding as the number and complexity of objectives increase. Among evolutionary approaches, the Non-Dominated Sorting Genetic Algorithm II (NSGA-II), introduced by Deb et al. (2002), has become one of the most widely used algorithms for multi-objective optimization. NSGA-II employs fast nondominated sorting and a crowding-distance mechanism to preserve diversity among candidate solutions while progressively moving the population toward the Pareto-optimal region. Its relatively simple structure, ability to handle nonlinear and non-convex search spaces, and capacity to generate multiple solutions in a single optimization run have contributed to its widespread adoption. Ramesh et al. (2012), for instance, demonstrated that an enhanced version of NSGA-II could effectively solve multi-objective reactive power planning problems and produce competitive Pareto solutions.

Despite these advantages, evolutionary algorithms such as NSGA-II are generally approximate or population-based methods and do not necessarily guarantee that every solution generated is an exact efficient solution of the underlying mathematical programming problem. Their performance can also depend on algorithmic parameters, population size, number of generations, crossover and mutation operators, and random initialization. Alexakis et al. (2025) identified computational time, scalability, model sensitivity, and formulation complexity among the continuing challenges associated with genetic algorithm-based optimization. These limitations are particularly relevant when evolutionary algorithms are applied to linear optimization problems for which mathematical programming methods may exploit the underlying structure of the problem more directly.

An alternative class of approaches for solving linear programming problems is provided by interior-point methods. Interior-point algorithms differ fundamentally from population-based evolutionary algorithms because they search through the interior of the feasible region rather than exploring a population of candidate solutions. Since the pioneering work of Karmarkar (1984), interior-point methods have become important alternatives to simplex-based methods for linear programming, particularly for large-scale optimization problems. Affine-scaling methods, in particular, iteratively move toward an optimal solution by scaling the feasible region around the current interior point. Their mathematical structure makes them attractive for linear optimization because they can exploit properties of the feasible region and objective functions in a manner that is fundamentally different from stochastic evolutionary search. For multi-objective linear programming, the problem is more challenging because the objective functions generally conflict and therefore produce a set of efficient solutions rather than one unique optimum. The fundamental objective is consequently to characterize the efficient frontier or generate a representative set of nondominated points. Classical approaches include scalarization techniques, weighted-sum methods, constraint-based approaches, and other interactive procedures in which the decision-maker's preferences are progressively incorporated into the optimization process. However, scalarization methods may not always provide an equally effective representation of the entire efficient frontier, particularly when the frontier possesses structural characteristics that limit the ability of a particular scalarization approach to recover all relevant efficient solutions.

Arbel's affine-scaling interior-point approach provides a mathematical framework for addressing MOLP through an interior-point strategy. The Affine Scaling Interior Multi-Objective Linear Programming (ASIMOLP) algorithm is designed to generate efficient or nondominated solutions by exploiting the structure of the feasible region while simultaneously considering multiple objectives. In contrast to NSGA-II, which relies on stochastic population evolution and nondominated sorting, ASIMOLP is based on a deterministic mathematical optimization framework. This fundamental difference raises an important methodological question: whether a specialized interior-point algorithm designed specifically for MOLP can produce higher-quality nondominated solutions than a widely used approximate evolutionary algorithm such as NSGA-II when both are applied to the same class of linear multi-objective problems.

Recent developments in multi-objective optimization provide further justification for such algorithmic comparisons. Zhu et al. (2026), for example, integrated surrogate modelling with NSGA-II to reduce computational costs in the multi-objective optimization of high-power permanent magnet synchronous motors. Their study showed that combining predictive models with evolutionary optimization can improve computational efficiency while retaining solution quality. Similarly, Zou et al. (2025) demonstrated that multi-objective optimization can effectively balance several competing building-performance criteria, while Becerra et al. (2026) showed that NSGA-II can be incorporated into hierarchical network-constrained optimization frameworks. These studies illustrate the versatility of NSGA-II but also emphasize that its effectiveness is often evaluated within specific application domains and against selected alternative approaches. Consequently, evidence concerning its relative performance against deterministic, problem-specific MOLP algorithms remains limited.

Although considerable research has been devoted to developing and applying multi-objective optimization algorithms, comparatively limited attention has been given to direct empirical comparisons between deterministic interior-point algorithms specifically designed for MOLP and general-purpose evolutionary algorithms such as NSGA-II. Existing studies have demonstrated the effectiveness of NSGA-II in diverse engineering, energy, building-design, and power-system applications (Alexakis et al., 2025; Becerra et al., 2026; Ramesh et al., 2012; Zou et al., 2025), while interior-point methods have a well-established theoretical foundation in mathematical programming (Karmarkar, 1984). However, the relative quality of the nondominated points produced by an affine-scaling interior-point MOLP algorithm and NSGA-II has not been sufficiently established through a systematic evaluation using common MOLP benchmark problems.

This gap is important because the quality of a multi-objective optimization method should not be assessed solely on whether it can generate feasible or nondominated solutions. The quality, convergence, distribution, and consistency of the resulting nondominated points are also critical considerations. A method that provides solutions closer to the true efficient frontier, with better distribution and greater consistency across repeated runs, may offer substantial advantages to decision-makers. Furthermore, stochastic and deterministic algorithms have fundamentally different search mechanisms, making direct comparison particularly valuable for determining the circumstances under which one approach may outperform the other.

The main aim of this study is to comparatively evaluate the performance of Arbel's Affine Scaling Interior Point Multi-Objective Linear Programming Algorithm (ASIMOLP) and the Non-Dominated Sorting Genetic Algorithm II (NSGA-II) in solving Multi-Objective Linear Programming problems.

Specifically, the study seeks to:

1. implement the ASIMOLP algorithm for solving selected MOLP problems;
2. implement NSGA-II for the same set of MOLP problem instances;
3. compare the nondominated points generated by ASIMOLP and NSGA-II;
4. evaluate the quality, convergence, distribution, and consistency of the solutions produced by the two algorithms; and
5. determine which algorithm provides superior nondominated solutions across a representative set of small- to medium-sized MOLP instances.

This study is significant both theoretically and computationally. From a methodological perspective, it provides an empirical basis for comparing a deterministic interior-point approach with a widely used evolutionary multi-objective optimization algorithm within the specific context of MOLP. Such a comparison can improve understanding of the relative strengths and limitations of mathematical-programming and evolutionary approaches to generating efficient solutions.

From a practical perspective, identifying the algorithm that consistently produces higher-quality nondominated points can assist researchers and practitioners in selecting appropriate optimization methods for multi-criteria decision problems. The findings may be particularly useful in applications involving resource allocation, energy planning, engineering design, transportation, and other areas where several conflicting linear objectives must be optimized simultaneously. The study also provides benchmark evidence that may support future research into hybrid optimization methods combining the mathematical rigor of interior-point algorithms with the flexibility and global-search capabilities of evolutionary algorithms.

The study therefore contributes to the growing body of research on multi-objective optimization by providing a systematic comparison of ASIMOLP and NSGA-II under a common experimental setting. By applying both algorithms to the same group of MOLP instances and evaluating the quality of their resulting nondominated points, the study provides evidence-based insight into the relative effectiveness of deterministic and evolutionary approaches for multi-objective linear optimization.

1.1 The General MOLP problem

The general MOLP problem as stated in Nyiam (2019) consist of q linear objective functions defined on X as follows:

$$
\begin{aligned}
f_1(x) &= c_{11}x_1 + c_{12}x_2 + \cdots + c_{1n}x_n \\
&\vdots \\
f_2(x) &= c_{21}x_1 + c_{22}x_2 + \cdots + c_{2n}x_{n,} \\
&\vdots \\
f_q(x) &= c_{q1}x_1 + c_{q2}x_2 + \cdots + c_{qn}x_{n,}
\end{aligned}
$$

Or

$$f_i(x) = \sum_{j=1}^{n} c_{ij}\, x_{j,}\ i = 1, \dots, q,$$

which are to be optimized subject to a set of m linear constraints:

$$
\begin{aligned}
g_1(x) &= a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le b_1 \\
g_2(x) &= a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le b_2 \\
&\vdots \\
g_m(x) &= a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le b_{m,}
\end{aligned}
$$

Or

$$g_r(x) = \sum_{j=1}^{n} a_{rj}\, x_j \le b_{r,} \quad r = 1, \dots, m.$$

These linear inequalities together with the non-negativity conditions $x_j \ge 0$ for all j forms the feasible set of solutions X. To “optimize” means either “minimize” or "maximize" the functions $f_i(x)$ over a nonempty convex polyhedron X which amounts to finding a vector $\hat{x} \in X$ such that

$$\sum_{j=1}^{n} c_{ij}\, x_j \leq \sum_{j=1}^{n} c_{ij}\, \hat{x}_j, \ or\ \sum_{j=1}^{n} c_{ij}\, x_j \geq \sum_{j=1}^{n} c_{ij}\, \hat{x}_j \text{ with } \sum_{j=1}^{n} c_{ij}\, x_j \neq \sum_{j=1}^{n} c_{ij}\, \hat{x}_j.$$

Coefficients c_{ij} can be positive, negative, or zero; they indicate the amount of gain (or loss) to be realized with respect to the i^{th} objective per each unit of increase in the j^{th}variable. Coefficients a_{rj} indicate how much of the r^{th} resource must be expanded per each unit of increase in x_j. Nyiam (2019).

The above general MOLP problem can be written in compact form as

$$\text{`Max' or `Min'} \qquad \begin{aligned} c_1{}^T x &= f_1 \\ c_2{}^T x &= f_2 \\ &\vdots \\ c_q{}^T x &= f_q \end{aligned} \qquad 1$$

Subject to $x \in X = \{x \in \mathbb{R}^n : Ax \leq b, b \in \mathbb{R}^m, x \geq 0\}$.

Or alternatively, as a linear vector optimization problem in matrix form as

$$\begin{aligned} \text{`Max' or `Min'} \quad & Cx \\ \text{Subject to} \quad & Ax = b \qquad 2 \\ & X \geq 0 \end{aligned}$$

where C is a $q \times n$ criterion matrix consisting of the rows $c_{k,}\ k = 1, \dots, q$; A is an $m \times n$ constraint matrix, $b \in \mathbb{R}^m$ is the right hand side vector and "min or max" amount to finding an element $\hat{x} \in X$ such that the solution $\hat{x}$ is not worst that any other solution but in no way the best, that is, the point $C\hat{x}$ cannot be smaller than or equal to all point $Cx, x \in X$, Nyiam & Salhi (2021a).

The feasible set in the decision space is $x \in X = \{x \in \mathbb{R}^n : Ax \leq b, b \in \mathbb{R}^m, x \geq 0\}$ and in the objective space it is $Y = Cx : x \in X$. The set Y is also referred to as the image of X. The nondominated faces in the objective space of the problem constitute the nondominated frontier while the efficient faces in the decision space of the problem constitute the efficient frontier, Salhi & Nyiam (2023).

A nondominated point in the objective space is the image of an efficient solution in the decision space, and the set of all nondominated points forms the nondominated set, Ehrgott etal. (2011).

An efficient solution of the problem is a solution that cannot improve any of the objective functions without deteriorating at least one of the other objectives.

Let $\hat{x} \in X$ be a feasible solution of (2) and let $\hat{y} = C\hat{x}$:

1. $\hat{x}$ is called efficient if there is no other $x \in X$ such that $Cx \leq C\hat{x}$ and $Cx \neq C\hat{x}$; correspondingly, $\hat{y} = C\hat{x}$ is called nondominated point.
2. $\hat{x}$ is called weakly efficient if there is no other $x \in X$ such that $Cx < C\hat{x}$; and $\hat{y} = C\hat{x}$ is called weakly nondominated point, Ehrgott (2006).

The set of all efficient solutions and the set of all weakly efficient solutions of (2) are denoted by X_E and X_{WE} respectively. $Y_N = \{Cx : x \in X_E\}$ and $Y_{WN} = \{Cx : x \in X_{WE}\}$ are the non-dominated and weakly non-dominated sets of (2) respectively, Benson (1998a).

1.2 Review of Related Literature

Multi-objective linear programming (MOLP) has attracted considerable research attention because of its ability to represent decision problems involving multiple, often conflicting, objectives subject to linear constraints. Unlike single-objective linear programming, in which a single optimal solution is generally sought, MOLP seeks efficient or nondominated solutions that represent different trade-offs among the competing objectives. Consequently, several exact, interactive, heuristic, and metaheuristic approaches have been developed to generate efficient solutions or useful approximations to the nondominated frontier (Ehrgott, 2006; Nyiam, 2019).

Early developments in interior-point approaches to MOLP were strongly influenced by adaptations of affine-scaling methods for single-objective linear programming. Arbel and Oren (1986) developed an approach based on the Affine Scaling Primal Algorithm (ASPA) for generating combined search directions in multi-objective problems. The method assumes an implicitly defined decision-maker utility function and constructs search directions that approximate the gradient of this utility function. The resulting directions are combined to generate successive interior iterates until an efficient solution is obtained. Building on this line of research, Arbel (1993a) developed the Affine Scaling Interior Multi-Objective Linear Programming (ASIMOLP) algorithm by adapting an affine-scaling variant of Karmarkar's interior-point approach. The algorithm generates individual search directions associated with the objective functions and combines them through a preference mechanism to obtain a common direction for moving through the feasible region. The procedure preserves interior feasibility while progressively approaching the efficient frontier. This approach provides an important alternative to simplex-based methods because it searches through the interior of the feasible region rather than moving primarily along its boundary.

Arbel (1993b) subsequently proposed another ASIMOLP formulation in which the Analytic Hierarchy Process (AHP) was used to derive preference information for combining the individual objective directions. The AHP-based mechanism enables the relative importance of the objectives to be incorporated explicitly into the search process. Arbel (1994) further extended this line of research by applying AHP-derived preference information to projected gradients to generate anchoring points and cones of opportunities for identifying preferred efficient solutions. These developments demonstrated the potential of interior-point methods to incorporate decision-maker preferences directly into the solution process.

Other researchers subsequently extended and modified the interior-point framework. Arbel and Korhonen (2001) introduced a starting mechanism that allows the algorithm to begin from either a feasible or infeasible point. Their approach augments the original problem and employs a nonnegative parameter to facilitate the search for an efficient solution. Zhong and Shi (2001) applied ASIMOLP to multiple-criteria and multiple-constraint-level problems by decomposing the problem into several MOLP subproblems and combining their efficient solutions through convex combinations. Lin et al. (2006) also modified an ASIMOLP approach by incorporating utility-based trade-offs among objectives and comparing the resulting procedure with earlier interior-point and simplex-based methods. Their computational experiments indicated that interior-point approaches can provide advantages in computational efficiency, particularly for relatively large problems.

Fliege (2006) proposed an approximate MOLP solution procedure based on a warm-start strategy and a path-following primal interior-point algorithm. Rather than repeatedly solving scalarized problems from independent starting points, the method uses information obtained from previously solved scalar problems to construct starting points for subsequent problems. Arbel and Sadka (2005), in another contribution to the interior-point literature, investigated the Euclidean center and its weighted version as a mechanism for exploring the efficient frontier. By varying the weights assigned to decision variables, different portions of the efficient frontier can be investigated.

Further developments have addressed specific computational limitations of interior-point MOLP algorithms. Wen and Weng (1998) modified an earlier interior-point procedure to reduce zigzagging during the search process, although their approach could, under some circumstances, terminate at an inefficient solution. Weng and Wen also developed an interior-point procedure based on weighted combinations of search directions and projection techniques for generating anchoring points. These studies demonstrate the continuing effort to improve the stability, convergence, and computational efficiency of interior-point approaches for MOLP.

More recently, Nyiam and Salhi (2021) developed an extended multi-objective simplex algorithm for solving MOLP problems, while Salhi and Nyiam (2023) investigated the application of the multi-objective plant propagation algorithm (MOPPA) and NSGA-II to MOLP. Their computational experiments involving 51 MOLP instances showed that MOPPA could perform competitively with established exact approaches, while NSGA-II produced well-distributed approximations across the nondominated frontier. These findings are particularly relevant because they demonstrate that population-based metaheuristics can be applied directly to MOLP and can provide useful approximations to the efficient frontier.

The application of evolutionary and metaheuristic techniques to multi-objective optimization has expanded considerably because exact approaches can become computationally demanding for complex problems. In worst-case situations, an MOLP problem may possess a very large number of efficient vertices, making complete enumeration computationally expensive (Khachiyan, 2008; Nyiam, 2019). Consequently, although exact methods provide theoretically rigorous solutions, approximate methods may be preferable when computational time, problem size, or practical decision-making requirements limit exhaustive enumeration.

One of the most influential evolutionary approaches is the Non-dominated Sorting Genetic Algorithm (NSGA) introduced by Srinivas and Deb (1995). The method uses nondominated sorting and genetic operators to evolve a population toward the Pareto frontier. Deb et al. (2002) subsequently introduced NSGA-II as an improved version of NSGA. NSGA-II incorporates an efficient nondominated sorting procedure, elitism, and crowding-distance estimation to maintain diversity among solutions. These characteristics have made NSGA-II one of the most widely used evolutionary algorithms for multi-objective optimization.

The use of evolutionary algorithms in MOLP has also been investigated in several application areas. Chakraborty (2010), for example, applied a multi-objective parametric fuzzy programming approach together with NSGA to an MOLP transportation problem. The formulation transformed the original problem into a parametric single-objective representation and demonstrated the applicability of evolutionary optimization to resource allocation. Similarly, Kannan et al. (2009) employed NSGA-II in energy-generation expansion planning to approximate competing investment and outage-cost objectives. These applications demonstrate the flexibility of evolutionary methods in addressing practical multi-objective decision problems.

Other metaheuristic approaches have also been investigated. Sanaz et al. (2006) proposed a linear multi-objective particle swarm optimization (LMOPSO) method for linearly constrained multi-objective optimization. The approach was designed to preserve feasibility while exploring the polyhedral feasible region and was demonstrated using a real-world chemical-system problem. Reyes-Sierra and Coello Coello (2006) reviewed multi-objective particle swarm optimization (MOPSO) methods and classified them into aggregation, lexicographic, subpopulation, Pareto-based, and hybrid approaches. Their review emphasized the growing importance of population-based optimization techniques in multi-objective decision-making and identified opportunities for further theoretical and practical development.

Zhang et al. (2020) proposed a multi-objective beetle antenna search algorithm (MOBAS) and evaluated it against established multi-objective algorithms, including MOPSO, NSGA-II, and multi-objective differential evolution. Their results demonstrated the potential of nature-inspired algorithms to provide competitive solutions for multi-objective optimization problems. Motahari et al. (2023) developed an MOLP model for a multi-product production system involving weighted completion time, transportation cost, and machine idle time. The e-constraint method was employed for smaller instances, whereas NSGA-II was used for larger and more realistic instances, illustrating the usefulness of evolutionary approaches when problem size increases.

The application of MOLP and evolutionary optimization has also expanded into contemporary engineering and energy-planning problems. Ramesh et al. (2012) applied a modified NSGA-II (MNSGA-II) to multi-objective reactive power planning, simultaneously minimizing operating and reactive-power allocation costs while improving voltage profiles and voltage stability. The proposed dynamic crowding-distance mechanism improved the diversity of nondominated solutions, and the method was tested on the IEEE 30-bus, practical 69-bus Indian, and IEEE 118-bus systems. The study demonstrates how modifications to NSGA-II can improve the distribution and quality of Pareto solutions in complex engineering optimization problems.

Recent studies further demonstrate the continuing development of multi-objective optimization methods. Hashemi et al. (2025) proposed an integrated data-driven framework for the multi-objective optimization of building forms at the early design stage, highlighting the increasing use of data-driven approaches for simultaneously addressing competing performance objectives. Similarly, Alexakis et al. (2025), in a systematic review of 175 studies, reported that NSGA-II remains one of the most frequently employed genetic algorithms for multi-objective building-retrofit optimization. Their review also identified persistent challenges involving computational time, data availability, occupant preferences, indoor environmental quality, climate sensitivity, and scalability.

Zou et al. (2025) applied multi-objective optimization to rural residential building design, simultaneously considering energy consumption, thermal comfort, and daylight performance. Their optimized designs achieved substantial improvements across all three objectives, demonstrating the practical value of multi-objective optimization when several performance criteria must be considered simultaneously. Becerra et al. (2026) recently employed NSGA-II within a hierarchical framework for the optimal siting and sizing of battery energy storage systems in the Mali transmission network. Their approach combined Pareto-based optimization with network-constrained power-flow analysis to balance economic returns against transmission losses. Similarly, Katkar and Jadhav (2026) proposed a hybrid metaheuristic framework for evaluating techno-economic and environmental performance in renewable-integrated power-flow optimization. Zhu et al. (2026) combined surrogate modelling with NSGA-II for the multi-objective optimization of high-power permanent-magnet synchronous motors, demonstrating how surrogate models can reduce computational burden while maintaining accurate optimization results.

The growing use of NSGA-II is therefore attributable to its ability to simultaneously generate multiple trade-off solutions, preserve population diversity, and avoid the need to specify a single set of objective weights before optimization. However, the method is fundamentally an approximate evolutionary approach and does not necessarily guarantee the complete recovery of the true efficient set of an MOLP problem. Its performance may also depend on population size, genetic operators, parameter settings, initialization, and the number of generations. In contrast, interior-point methods such as ASIMOLP exploit the mathematical structure of linear constraints and objectives and can provide efficient solutions through a systematic search within the feasible region.

Nyiam and Salhi (2026) subsequently modified ASIMOLP to address zigzagging during the search process. Their modification introduced an equal-weight generating function whose components were used to construct a convex combination of individual search directions. The resulting compromise direction enabled the algorithm to move more effectively toward the efficient frontier and improved convergence within an acceptable number of iterations. These developments indicate that ASIMOLP remains an active area of research and that improvements to its search mechanism can enhance its computational performance.

Despite these advances, a clear methodological gap remains. Previous studies have separately examined the computational performance of interior-point methods, simplex-based exact approaches, and evolutionary algorithms, while relatively limited attention has been given to a direct and systematic comparison of the quality of nondominated solutions generated by ASIMOLP and NSGA-II on the same MOLP problem instances under comparable computational conditions. Existing studies involving NSGA-II have largely focused on nonlinear, engineering, scheduling, energy, transportation, and other application-specific problems, whereas relatively few have specifically evaluated its performance against an interior-point MOLP algorithm using linear benchmark instances. Moreover, computational comparisons often emphasize either CPU time or general algorithmic performance without sufficiently examining the objective values of the resulting nondominated solutions.

The present study addresses this gap by implementing Arbel's ASIMOLP and NSGA-II and applying both algorithms to the same collection of MOLP instances ranging from small to medium dimensions. The comparison is conducted under a common computational environment, with particular emphasis on the quality of the nondominated solutions returned by the two algorithms. This approach provides a more direct assessment of whether an exact interior-point strategy such as ASIMOLP provides superior nondominated objective values compared with an evolutionary approximation method such as NSGA-II when both are applied to linear multi-objective problems.

The literature therefore suggests a complementary relationship between the two approaches. ASIMOLP benefits from the mathematical structure of MOLP and its interior-point search mechanism, whereas NSGA-II benefits from population-based exploration, elitism, and diversity preservation. A systematic comparison of their solution quality can provide useful evidence for researchers and practitioners in selecting appropriate algorithms for MOLP problems. The findings may also contribute to the continuing development of hybrid and improved MOLP algorithms by identifying the relative strengths and limitations of exact interior-point and approximate evolutionary approaches.

Algorithm 1 Affine Scaling Interior MOLP Algorithm Nyiam & Salhi (2021)

0: Input: A, b, C : *Problem data*
1: Initialize: *Choose* $x_0 > 0$, *Stopping tolerance* σ $(0 < \sigma < 1)$, *Step size factor* ρ $(0 < \rho < 1)$, *Converged* $= 0$, $k \leftarrow 0$;
2: while *Converged* $\neq 1$ **do**
3: $k \leftarrow k + 1$
4: $D \leftarrow diag(x_0)$
5: $y_i(k) \leftarrow (AD^2A^T)^{-1}AD^2c_i$
6: $dx_i(k) \leftarrow D^2(c^T - A^T y_i(k))$
7: if $dx_i(k) \geq 0,\ 1 \leq i \leq q$, **stop**
8: **else**
9: $\alpha_i \leftarrow Min[x_i/dx_i(k)\ \forall dx\ (k) < 0]$
10: $x_i(k+1) \leftarrow x(k) + \rho\alpha_i dx_i(k)$
11: $dx \leftarrow \sum p_i \alpha_i dx_i,\ 1 \leq i \leq q$
12: $x(k+1) \leftarrow x(k) + \rho dx(k)$
13: $x_{new} \leftarrow x(k) + dx(k)$
14: $v_{new} \leftarrow C^T x_{new}$
15: $dx_{end} \leftarrow x_{end} - x(k)$
16: **if** $k > 1$, **do**
17: $dx \leftarrow \sum p_i \alpha_i dx_i + p_{end} dx_{end}, 1 \leq i \leq q;$
18: $x_{new} \leftarrow x(k) + \rho dx(k)$
19: $v_{new} \leftarrow C^T x_{new}$
20: $x_{end} \leftarrow x(k) + dx(k)$
21: $v_{end} \leftarrow C^T x_{end}$
22: $dx_{end} \leftarrow x_{end} - x_{new}$
23: **else**
24: **if** $\|dx_{end}\| \leq \sigma$, **stop**
25: **else**
26: $x_0 \leftarrow x_{new}$
27: **Go to step 4**
28: **endif**
29: **endif**
30: **endif**
31: endwhile
32 Output: x_{end} *Efficient solution*
v_{end}: *objective values*

2.0 Nondominated Sorting Genetic Algorithm II

We now present the pseudo-code of Nondominated Sorting Genetic Algorithm II as adapted from Yuen & Ramli (2010).

Algorithm 2 Nondominated Sorting Genetic Algorithm II, Yuen & Ramli (2010)

1: Initialization:
2: ⋄ Generate random population
3: ⋄ Evaluate objective values
4: ⋄ Assign rank (level) based on nondomination
5: ⋄ Generate child population
- Tournament selection
- Crossover and mutation

For i = 1 to No. of generations
6: ⋄ Parent & child population are assign rank based on nondomination
7: ⋄ Generate sets of nondominated fronts
8: ⋄ Determine the crowding distance between points on each front
9: ⋄ Select points based on crowding distance calculation & fill into the parent population until full
10: ⋄ Create next generation
11: ⋄ Tournament Selection
12: ⋄ Crossover and Mutation
13: ⋄ Evaluate Objective Values
14: ⋄ Increment generation index
End

3.0 Experimental Results

This section presents the numerical results obtained from the application of Arbel's Affine Scaling Interior Multi-Objective Linear Programming Algorithm (ASIMOLP) and the Nondominated Sorting Genetic Algorithm II (NSGA-II) to ten MOLP benchmark instances. The test problems comprise small- to medium-sized instances obtained from the literature and randomly generated problems with different dimensions and matrix structures. The problems were selected to provide variation in the number of decision variables, constraints, and objective functions and thereby permit a comparative assessment of the performance of the two algorithms.

Algorithm 1, representing ASIMOLP, was implemented in MATLAB based on the procedure described by Arbel (1993a) and adapted by Nyiam and Salhi (2021). Algorithm 2, representing NSGA-II, was implemented using a MATLAB-based formulation adapted from Yuen and Ramli (2010). For each problem, (n) denotes the number of decision variables, (m) denotes the number of constraints, and (q) denotes the number of objective functions. The algorithms were applied to identical problem formulations to ensure that the resulting solutions could be compared under the same optimization conditions.

Table 1 presents the most-preferred solutions obtained from ASIMOLP and NSGA-II for the ten test problems. Problems 1–4 were obtained from established sources, while Problems 5–10 were generated to provide additional small- to medium-scale test instances. The benchmark dimensions range from 2 to 35 decision variables and from 2 to 60 constraints, with two- or three-objective formulations. This range provides an initial assessment of the behaviour of the algorithms as problem size and structural complexity increase.

Table 1: Comparative results for individual problem

	Algorithms				ASIMOLP	NSGA-II
Origin	**Prob.**	**N**	**m**	**Q**	**Most preferred Solution**	**Most preferred Solution**
Ehrgott 2006	1	3	3	3	f1 = -1.74 f2 = 5.56 f3 = -2.75	f1 = -0.35 f2 = 3.27 f3 = -1.98

Zeleny 1982	2	2	2	2	f1 = -30626 f2 = -64132	f1 = -22302 f2 = -34750
"	3	2	4	3	f1 = -3.50 f2 = -2.74 f3 = 4.89	f1 = -3.10 f2 = -2.50 f3 = -4.13
"	4	7	4	3	f1 = -7.65 f2 = -13.80 f3 = -7.75	f1 = -6.89 f2 = -11.15 f3 = -2.18
Randomly generated with dense matrices	5	30	15	3	f1 = -247.63 f2 = -208.93 f3 = -270.71	f1 = -170.41 f2 = -189.70 f3 = -246.81
"	6	35	20	3	f1 = -163.40 f2 = -265.94 f3 = -266.47	f1 = -121.27 f2 = -241.10 f3 = -211.15
"	7	20	40	3	f1 = -72.23 f2 = -63.77 f3 = -41.14	f1 = -45.30 f2 = -40.20 f3 = -34.10
Randomly generated with dense matrices	8	15	45	3	f1 = -39.52 f2 = -93.22 f3 = -92.02	f1 = -24.40 f2 = -53.90 f3 = -70.80
"	9	30	50	3	f1 = -56.74 f2 = -68.66 f3 = -65.62	f1 = -21.94 f2 = -50.02 f3 = -56.56
"	10	30	60	3	f1 = -23.12 f2 = -69.40 f3 = -203.25	f1 = -15.15 f2 = -44.62 f3 = -183.41

The results in Table 1 indicate noticeable differences between the solutions obtained by the two algorithms. For Problem 1, ASIMOLP produced the objective values (f_1=-1.74), (f_2=5.56), and (f_3=-2.75), whereas NSGA-II produced (f_1=-0.35), (f_2=3.27), and (f_3=-1.98). Similar differences are observed for Problems 2–10. In several instances, the ASIMOLP solution provides more favourable objective-function values than the corresponding NSGA-II solution, indicating stronger performance for the tested problem formulations. However, the interpretation of individual objective values must be made in accordance with the specified maximization or minimization direction of each objective. For the randomly generated problems, the difference between the two approaches becomes more apparent as the dimensions of the problem increase. For example, in Problems 5 and 6, ASIMOLP produced objective values of ((-247.63,-208.93,-270.71)) and ((-163.40,-265.94,-266.47)), respectively, compared with ((-170.41,-189.70,-246.81)) and ((-121.27,-241.10,-211.15)) obtained using NSGA-II. Similar differences are observed for Problems 7–10. These results indicate that ASIMOLP retained competitive performance as the number of variables and constraints increased within the range investigated.

The observed differences can be attributed partly to the fundamentally different search mechanisms of the two algorithms. ASIMOLP exploits the geometry and linear structure of the feasible region and constructs search directions directly from the objective functions and constraint system. Consequently, its search is closely associated with the mathematical structure of the MOLP problem. NSGA-II, in contrast, employs a population-based stochastic search in which candidate solutions evolve through selection, crossover, mutation, nondominated sorting, and crowding-distance preservation. While this mechanism provides considerable flexibility and allows NSGA-II to approximate a diverse set of nondominated solutions, its performance may be influenced by population size, genetic parameters, initialization, and random variation.

The results therefore suggest that the specialized structure of ASIMOLP may provide an advantage for the tested linear multi-objective problems. In particular, the relatively favourable objective values obtained by ASIMOLP across most of the benchmark instances indicate that a deterministic interior-point approach can be highly competitive with an evolutionary approximation method when the underlying optimization problem possesses a linear structure.

Nevertheless, the results should be interpreted with some caution. The present comparison is based on ten MOLP instances and the reported objective values do not, by themselves, provide a complete assessment of Pareto-front quality, diversity, or statistical robustness. Furthermore, because NSGA-II is stochastic, a single computational run may not fully represent its performance. A more comprehensive comparison would require multiple independent NSGA-II runs and the use of established multi-objective performance indicators, including hypervolume, inverted generational distance, generational distance, spacing, number of nondominated solutions, and computational time.

Overall, the experimental results provide preliminary evidence that ASIMOLP can produce highly competitive solutions and, for most of the test problems considered, more favourable objective-function values than NSGA-II. The findings support the continued investigation of interior-point approaches for MOLP and provide a basis for future comparisons involving larger benchmark collections, repeated stochastic runs, standardized Pareto-front quality measures, and hybrid approaches combining the strengths of deterministic and evolutionary optimization techniques.

4.0 Conclusion

This study comparatively evaluated Arbel’s Affine Scaling Interior Multi-Objective Linear Programming Algorithm (ASIMOLP) and the Nondominated Sorting Genetic Algorithm II (NSGA-II) for solving multi-objective linear programming (MOLP) problems. The two approaches represent fundamentally different optimization paradigms: ASIMOLP exploits the mathematical structure of the feasible region through an interior-point search, whereas NSGA-II uses evolutionary search, nondominated sorting, and crowding-distance mechanisms to approximate the Pareto frontier.

Numerical experiments involving ten small- to medium-sized MOLP instances demonstrated that ASIMOLP generally produced more favourable objective-function values than NSGA-II for most of the problems considered. The results suggest that, for the tested MOLP instances, the problem-specific interior-point strategy of ASIMOLP can provide competitive and, in several cases, superior solutions compared with the approximate evolutionary approach. This finding highlights the potential advantage of exploiting the linear structure of MOLP rather than relying exclusively on stochastic population-based search. However, the results should not be interpreted as establishing the universal superiority of ASIMOLP over NSGA-II. NSGA-II is stochastic and its performance can depend on population size, genetic operators, parameter settings, and random initialization. Moreover, the present study is based on a relatively limited number of benchmark instances. Future research should therefore employ larger and more diverse MOLP benchmark sets, multiple independent NSGA-II runs, standardized performance indicators such as hypervolume, inverted generational distance, spacing, and computational time, and appropriate statistical tests.

Finally, the study provides useful comparative evidence on the application of deterministic interior-point and evolutionary approaches to MOLP. The findings establish a basis for further investigation of hybrid methods that combine the mathematical efficiency and convergence properties of ASIMOLP with the population-based exploration and diversity-preservation capabilities of NSGA-II.

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Declarations

Ethics Approval and Consent to Participate

This study utilized publicly available and anonymized CT image datasets. Therefore, ethical approval and informed consent were not required.

Consent for Publication

All authors have read and approved the final manuscript and consented to its publication.

Availability of Data and Materials

The datasets and materials used in this study are available from publicly accessible sources. Additional information may be obtained from the corresponding author upon reasonable request.

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Competing Interests

The authors declare that they have no competing financial or personal interests that could have influenced the work reported in this study.

Author Contributions

All components of the work were carried out by the author.