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Application of Objective-Space and Multi-Objective Plant Propagation Algorithms to Multi-Objective Linear Programming

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Abstract: *The solution of Multiple Objective Linear Programming (MOLP) problem is usually obtained by exact methods. However, nature-inspired population-based algorithms such as the multi-objective plant propagation algorithm (MOPPA) is becoming more and more prominent. This paper presents an application of the multiple-objective plant propagation algorithm (MOPPA) and an Objective space-based method known as Benson's outer approximation algorithm to multi-objective linear programming in order to compare the quality of nondominated points returned by these two Algorithms. Computational results for a selected group of 20 MOLP instances ranging from small to realistic instances was reported. It was observed that Benson's outer approximation algorithm is superior to MOPPA in terms of the quality of nondominated points it returned.*

Keywords: *Multi-Objective Linear Programming; Plant Propagation Algorithm; Benson's Outer Approximation Algorithm; Nondominated Solutions.*

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1, 0 Introduction

Multi-Objective Linear Programming (MOLP) is an important branch of mathematical programming concerned with decision-making problems in which two or more, often conflicting, linear objectives must be optimized simultaneously subject to a common set of constraints. Unlike single-objective linear programming, where an optimal solution can generally be identified with respect to one objective function, MOLP requires the decision maker to consider trade-offs among competing objectives. This makes MOLP particularly relevant to complex decision-making problems in engineering, management, production planning, logistics, resource allocation, finance, energy systems, and other application areas where several performance criteria must be considered simultaneously. The increasing complexity of real-world decision environments has consequently strengthened the importance of multi-objective optimization as a framework for identifying and evaluating alternative compromise solutions (Cerda-Flores et al., 2022; Karimi et al., 2022).

A general MOLP problem can be written as a linear vector optimization problem:

$$\begin{aligned} &\min Cx \\ \text{subject to}\quad &Ax = b \\ &x \geq 0 \end{aligned} \tag{1}$$

where C is a $q \times n$ criterion matrix consisting of the rows C_k, $k = 1, .., q$, A is an $m \times n$ constraint matrix and the right hand side vector $b \in \mathbb{R}^m$. The feasible set in the decision space is $X = \{x \in \mathbb{R}^n : Ax = b, x \geq 0\}$ and the feasible set in the objective space is $Y = \{Cx : x \in X\}$.

Because the objectives in an MOLP problem may conflict, it is generally impossible to optimize all objective functions simultaneously. Consequently, the conventional concept of a single optimal solution is replaced by the concepts of efficient solutions in the decision space and nondominated points in the objective space. An efficient solution is a feasible solution for which an improvement in one objective cannot be achieved without deterioration in at least one other objective. The corresponding image of an efficient solution in the objective space is referred to as a nondominated point. Thus, the principal purpose of MOLP is to identify the efficient set or its corresponding nondominated set, or alternatively to obtain a preferred solution from among the available efficient alternatives (Nyiam & Salhi, 2023).

We also stated in Nyiam & Salhi (2026) that an efficient solution of a MOLP problem is a solution that cannot improve some of the objective functions without deteriorating at least one of the other objectives, while a weakly efficient solution of an MOLP is a solution that cannot improve all the objective functions at the same time. A nondominated point in the objective space is the image of an efficient solution in the decision space, and the set of all nondominated points forms the nondominated set in the objective space.

Mathematically, let $\hat{x} \in X$ be a feasible solution of (1) and let $\hat{y} = C\hat{x}$: $\hat{x}$ is called efficient if there is no $x \in X$ such that $Cx \leq C\hat{x}$ and $Cx \neq C\hat{x}$; correspondingly, $\hat{y} = C\hat{x}$ is called nondominated. $\hat{x}$ is called weakly efficient if there is no $x \in X$ such that $Cx < C\hat{x}$; and $\hat{y} = C\hat{x}$ is called weakly nondominated.

The set of all efficient solutions and the set of all weakly efficient solutions of (1) are denoted by X_E and X_{WE} respectively. X_E and X_{WE} are called the efficient decision set and the weakly efficient decision set of (1) respectively. The sets $Y_N = \{Cx{:}x\in X_E\}$ and $Y_{WN} = \{Cx : x\in X_{WE}\}$ are the nondominated and weakly nondominated sets in the objective space of (1) respectively, Nyiam & Salhi (2026).

MOLP has been an active research area for several decades, and numerous exact and approximate methods have been developed for generating efficient solutions and nondominated points. Classical approaches have traditionally relied on mathematical programming techniques capable of systematically exploring the feasible and objective spaces. Objective-space methods are particularly important because they directly characterize the geometry of the attainable objective set and can, under appropriate conditions, generate the complete nondominated frontier. One prominent example is Benson's outer-approximation algorithm (BOA), which operates in the objective space and progressively constructs an outer approximation of the upper image until the nondominated set is identified. Such objective-space approaches provide an attractive benchmark against which approximate or stochastic methods can be evaluated.

In contrast, the increasing complexity and dimensionality of contemporary optimization problems have encouraged the development of population-based and nature-inspired algorithms. Multi-objective optimization has been increasingly employed in practical applications because it provides a systematic means of examining trade-offs among conflicting performance measures (Cerda-Flores et al., 2022). Furthermore, uncertainty in real-world optimization problems has stimulated the development of more flexible multi-objective approaches, including fuzzy and other computational methods capable of accommodating complex decision environments (Karimi et al., 2022). Recent algorithmic developments have also continued to explore alternative strategies for improving the computational efficiency and quality of multi-objective solutions. For example, Olfati et al. (2025) proposed a goal-programming-based algorithm that sequentially establishes objective-specific targets and minimizes deviations from those targets, demonstrating the continuing interest in developing efficient approaches for multi-objective optimization.

Among nature-inspired optimization techniques, the Plant Propagation Algorithm (PPA) has attracted attention because of its population-based search mechanism inspired by the growth and reproduction characteristics of plants. The extension of PPA to multi-objective problems, known as the Multi-Objective Plant Propagation Algorithm (MOPPA), provides an alternative approach for approximating efficient solutions and nondominated points. Unlike objective-space exact methods, MOPPA primarily operates through population-based search in the decision space and subsequently evaluates the corresponding objective vectors. Its stochastic nature allows it to explore large and complex solution spaces and potentially provide good approximations within reasonable computational effort.

Nyiam and Salhi (2023) demonstrated the applicability of MOPPA to MOLP problems and compared its performance with the Nondominated Sorting Genetic Algorithm II (NSGA-II) and four established exact methods: the Extended Multiple Objective Simplex Algorithm (EMSA), Affine Scaling Interior MOLP Algorithm (ASIMOLP), Benson's Outer-Approximation Algorithm (BOA), and Parametric Simplex Algorithm (PSA). Using 51 existing MOLP instances, they reported that MOPPA compared favourably with the exact methods, particularly for large instances that some exact approaches were unable to solve within the allotted computational time. Their study also showed that MOPPA generated nondominated points of higher quality than NSGA-II, although NSGA-II produced a relatively even distribution of points along the nondominated front. These findings demonstrate the potential of MOPPA as an alternative computational approach to MOLP.

Despite these developments, a methodological distinction remains between algorithms that approximate the nondominated set through stochastic population-based search and exact objective-space algorithms designed to systematically determine the nondominated frontier. The quality of the solutions generated by these fundamentally different approaches cannot be assessed solely from their ability to produce feasible or nondominated points. Important considerations include the quality of the nondominated points, their proximity to the true nondominated frontier, completeness of the identified efficient set, and their consistency across problem instances. Although Nyiam and Salhi (2023) established the feasibility of applying MOPPA to MOLP and demonstrated encouraging results, a more focused comparison between MOPPA and an objective-space exact method such as Benson's outer-approximation algorithm remains necessary. In particular, there is a need to determine whether the computational advantages of a population-based stochastic approach are accompanied by a comparable quality of nondominated solutions when evaluated against an objective-space method capable of systematically identifying the nondominated set.

This constitutes the principal knowledge gap addressed by the present study. Existing literature has extensively examined multi-objective optimization methods and has established both exact mathematical programming approaches and nature-inspired algorithms as viable solution strategies. However, relatively limited attention has been devoted to a direct and systematic assessment of MOPPA against Benson's outer-approximation algorithm specifically in terms of the quality of the nondominated points obtained from MOLP instances of different sizes and levels of complexity. Addressing this gap is important because the choice between an exact objective-space algorithm and an approximate population-based method should be informed not only by computational considerations but also by the quality and reliability of the solutions produced.

Therefore, this study aims to apply the Multi-Objective Plant Propagation Algorithm (MOPPA) and Benson's Outer-Approximation Algorithm (BOA) to selected Multiple Objective Linear Programming problems and comparatively evaluate the quality of the nondominated points generated by the two approaches. Specifically, the study considers a collection of 20 MOLP instances ranging from small-scale to relatively realistic problems and examines the ability of the two algorithms to identify high-quality nondominated solutions.

The significance of the study lies in providing further empirical evidence on the relative effectiveness of stochastic population-based and objective-space approaches to MOLP. The findings will contribute to the growing literature on multi-objective optimization by clarifying the circumstances under which MOPPA can provide competitive solutions and the extent to which BOA provides superior nondominated points. The results may also assist researchers and practitioners in selecting appropriate optimization strategies according to problem size, solution quality requirements, and the need for exact or approximate representations of the nondominated frontier. More broadly, the study contributes to the development of reliable computational tools for multi-criteria decision-making in situations where conflicting objectives must be considered simultaneously.

The remainder of this paper is organized as follows. Section 2 presents a review of relevant literature on heuristic, population-based, and objective-space approaches to MOLP. Section 3 describes the Multi-Objective Plant Propagation Algorithm (MOPPA), while Section 4 presents Benson's Outer-Approximation Algorithm (BOA). Section 5 reports and discusses the computational results obtained from the selected MOLP instances and compares the quality of the nondominated points generated by the two algorithms. Finally, Section 6 presents the conclusions and implications of the study.

2.0 Review of related literature

Research on Multiple Objective Linear Programming (MOLP) has produced a wide range of solution strategies, reflecting the computational difficulty of simultaneously handling several conflicting objectives. The existing approaches can broadly be distinguished into exact mathematical programming methods and approximate or population-based methods. The former seek to systematically determine efficient solutions or the complete nondominated set, whereas the latter generally aim to obtain high-quality approximations within acceptable computational times. This distinction is particularly relevant when considering large or realistic MOLP instances for which exhaustive enumeration may become computationally demanding.

Multi-objective optimization has received considerable attention because of its relevance to decision-making problems involving competing performance criteria. Applications occur in industrial processes, resource allocation, production planning, logistics, engineering design, energy systems, and other areas in which improving one criterion may adversely affect another. Cerda-Flores et al. (2022) observed that multi-objective optimization provides an effective framework for analysing such trade-offs and has become an important decision-support tool in industrial applications. Similarly, Karimi et al. (2022), in their systematic review of fuzzy multi-objective programming, demonstrated the extensive development of multi-objective techniques for decision environments characterized by conflicting objectives and uncertainty. More recently, Olfati et al. (2025) proposed a goal-programming-based approach in which individual objective functions are first optimized to establish target values, after which deviations from these targets are minimized. These developments demonstrate the continuing search for computational approaches capable of obtaining satisfactory solutions to increasingly complex multi-objective problems.

2.1 Approximate and Population-Based Approaches to MOLP

Approximate methods have become increasingly attractive where obtaining the complete efficient or nondominated set using an exact procedure requires substantial computational resources. Population-based algorithms are particularly useful because they simultaneously examine multiple candidate solutions and can explore different regions of the search space. Their stochastic search mechanisms also make them suitable for problems where the solution landscape is complex or where obtaining an exact representation of the entire nondominated frontier is computationally expensive.

The application of nature-inspired algorithms to MOLP has received comparatively less attention than their application to nonlinear multi-objective optimization. One important development in this direction is the adaptation of the Plant Propagation Algorithm (PPA) to multi-objective optimization. The PPA is inspired by the propagation behaviour of plants, in which favourable locations encourage intensive local propagation while less favourable locations encourage longer-range exploration. This search mechanism provides a balance between exploitation of promising regions and exploration of alternative areas of the solution space.

Nyiam and Salhi (2023) extended this concept to the Multi-Objective Plant Propagation Algorithm (MOPPA**)** and applied it to MOLP problems. Their study compared MOPPA with the Nondominated Sorting Genetic Algorithm II (NSGA-II) and four established exact approaches: the Extended Multiple Objective Simplex Algorithm (EMSA), Affine Scaling Interior MOLP Algorithm (ASIMOLP), Benson's Outer-Approximation Algorithm (BOA), and the Parametric Simplex Algorithm (PSA). Using 51 existing MOLP instances, the authors reported that MOPPA compared favourably with the exact methods, particularly for relatively large instances for which some exact algorithms encountered computational difficulties. MOPPA also generated nondominated points of higher quality than NSGA-II, although NSGA-II produced a more evenly distributed approximation of the nondominated front. These findings established the feasibility of using population-based plant propagation as an alternative strategy for MOLP.

The findings of Nyiam and Salhi (2023) are important because they demonstrate that an approximate population-based algorithm can provide competitive solutions to MOLP without necessarily generating the complete nondominated set. However, the stochastic nature of MOPPA means that its performance depends on the search process and the characteristics of the problem instance.

Consequently, evaluating the quality of its generated nondominated points against an exact method remains important, particularly where the exact method is specifically designed to operate in objective space.

2.2 Objective-Space Approaches to MOLP

An alternative to searching primarily through the decision variables is to formulate the solution procedure around the geometry of the objective space. Objective-space methods exploit the structure of the attainable objective set and provide a systematic means of identifying nondominated points. Such approaches are particularly valuable when the objective of the analysis is to obtain a reliable representation of the nondominated frontier against which approximate solutions can be assessed.

Among the established objective-space approaches, Benson's Outer-Approximation Algorithm (BOA) is particularly important. The method progressively constructs an outer approximation of the relevant objective-space region and refines this approximation until the nondominated set is determined. In contrast to stochastic population-based methods, an objective-space exact procedure provides a systematic mechanism for identifying the nondominated structure of the MOLP problem. This makes BOA particularly suitable as a reference approach when assessing the quality of solutions obtained from approximate algorithms.

The distinction between these two methodological perspectives is important. MOPPA searches for promising solutions through a population-based mechanism in the decision space and subsequently evaluates their objective vectors. BOA, on the other hand, works directly with the geometry of the objective space and is designed to systematically determine nondominated solutions. Consequently, the two approaches differ not only in their computational procedures but also in the manner in which they represent and explore the solution space..

2.3 Comparative Evaluation of MOLP Solution Approaches

The literature indicates that no single class of algorithms is universally superior for every multi-objective optimization problem. Exact approaches provide systematic solution procedures and are particularly valuable where completeness and solution certainty are important. However, their computational requirements may become restrictive as problem size and complexity increase. Approximate population-based algorithms, in contrast, may provide good solutions with comparatively flexible computational requirements, but they do not necessarily guarantee complete recovery of the nondominated frontier.

This trade-off is consistent with the broader development of multi-objective optimization. Cerda-Flores et al. (2022) emphasized the importance of computational approaches that can accommodate multiple and sometimes competing performance measures in practical applications, while Karimi et al. (2022) demonstrated the continuing expansion of multi-objective methods for complex decision environments. The development of alternative algorithms, including the goal-programming strategy proposed by Olfati et al. (2025), further illustrates the continuing effort to improve the quality and efficiency of multi-objective solution procedures.

Within MOLP specifically, the study of Nyiam and Salhi (2023) provides evidence that MOPPA can generate competitive nondominated points. Nevertheless, their work considered MOPPA alongside several exact approaches rather than focusing specifically on a detailed comparison between MOPPA and an objective-space method in terms of the quality of the resulting nondominated points. A focused comparison is therefore warranted because the two approaches represent fundamentally different computational philosophies: one relies on stochastic population-based exploration, whereas the other systematically exploits the structure of the objective space.

Although previous research has established the usefulness of both population-based and objective-space approaches in multi-objective optimization, there remains a need for a direct assessment of their relative performance in MOLP. In particular, the evidence provided by Nyiam and Salhi (2023) confirms the potential of MOPPA, but does not by itself establish whether its approximated nondominated points can match the quality of those generated by an objective-space exact procedure such as BOA across MOLP instances of different sizes and complexity.

The present study therefore addresses this methodological gap by concentrating specifically on the comparative quality of the nondominated points obtained using MOPPA and BOA. The comparison is based on a selected collection of 20 MOLP instances ranging from small to relatively realistic problems. Such an assessment provides a basis for determining whether the flexibility of the population-based MOPPA approach is accompanied by solution quality comparable to that obtained from an objective-space exact algorithm.

3.0 Solution Algorithms

This study employs two fundamentally different approaches for solving Multiple Objective Linear Programming (MOLP) problems: the Multi-Objective Plant Propagation Algorithm (MOPPA), which is a population-based stochastic approach, and Benson's Outer-Approximation Algorithm (BOA), which operates in the objective space.

3.1 Multi-Objective Plant Propagation Algorithm (MOPPA)

The Multi-Objective Plant Propagation Algorithm (MOPPA) is a population-based stochastic optimization approach adapted for MOLP problems. The algorithm employs population generation, fitness-based selection, propagation through runners, pruning of similar solutions, and elitist retention of nondominated solutions. The pseudocode adopted in this study, based on Nyiam and Salhi (2023), is presented in Algorithm 1.

Algorithm 1: The Multi-Objective Plant Propagation Algorithm.

Given: *f(x)*; a vector function; n_g; number of generations to perform; n_p; the propagation size; n_r; maximum number of runners to propagate:
Output: *z*; vector approximation to Nondominated frontier:
    p ⟵ initial random population of size n_p
  for n_g generations **do**
        prune population p; removing similar solutions
        N ⟵ fitness(p), *Use rank-based fitness*
        Ṕ ⟵ ∅, Empty set
        for i ⟵ 0 … n_p **do**
        x ⟵ select(p;N), Tournament fitness based selection
        for each runner to gen. ***do*** No. proportional to fitness rounded up
          ẍ ⟵ new solution (x,1-N) Dist. inversely proportional to fitness
          Ṕ ⟵ ẍUṔ Add to new population
endfor

```
p ⟵ p\x, Remove from old population
        endfor
p ⟵ ṔUNondominated(p) New population with elitism
endfor
z ⟵ Nondominated(p)
```

3.2 Benson's Outer-Approximation Algorithm (BOA)

Benson's Outer-Approximation Algorithm (BOA) is an objective-space method for determining the nondominated set of a MOLP problem. Unlike MOPPA, which uses population-based stochastic search, BOA progressively constructs an outer approximation of the relevant objective-space region and refines this approximation through successive linear programming solutions. The pseudocode used in this study is adapted from Nyiam and Salhi (2023).

4. **Algorithm 2: The Benson's Outer Approximation Algorithm.**

```
Input: A, b, P
        a finitely generated solution ({0}, X^h) to P^h;
        a finitely generated solution T^h to D*^h;
Initialize: ^p ← P(solve(P1(0))) + e;
        T ← {(solve(D1(w)),w)|(u,w)∈T^h};
while z ≠ 0 do
    X^d ← {D*(u,w)|(u,w)∈T};
        X^p ← vert(X^d);
        ẍ ← ∅;
        for i = 1 to |X^p| do
        v ← Xp[i];
        (x, z) ← solve(P2(v));
ẍ ← ẍ∪{x};
if z > 0 then
  (x, δ) ← solve(R(v));
  y ← δv + (1− δ)^p;
  (u,w) ← solve(D2(y));
  T ← T∪{(u,w)};
  break;
endif;
    endfor;
endwhile
Output: (ẍ,X^h) a finitely generated solution to P;
        T a finitely generated solution to D*
```

3.3 Comparative Basis of the Algorithms

The two algorithms differ fundamentally in their solution strategies. MOPPA performs a population-based search and approximates the nondominated frontier through evolutionary propagation and selection, whereas BOA systematically explores the objective space to determine the nondominated structure of the MOLP problem. Consequently, the comparison provides an assessment of a stochastic approximate method against an objective-space exact approach. The principal basis of comparison in this study is the quality of the nondominated points generated by the two algorithms across the selected MOLP instances.

4.0 Experimental Results and Discussion

This section presents and discusses the computational results obtained from the Multi-Objective Plant Propagation Algorithm (MOPPA) and Benson's Outer Approximation Algorithm (BOA) for the selected MOLP test problems. The comparison focuses primarily on the quality of the nondominated points generated by the two algorithms across 20 MOLP instances of varying dimensions and levels of complexity.

4.1 Description of the Test Problems and Computational Procedure

Table 1 summarizes the numerical results obtained for the 20 selected MOLP test problems. The test set comprises small, medium-sized, and relatively realistic instances drawn from established sources and randomly generated datasets. Problem 1 was obtained from Ehrgott (2006), while Problems 2–4 were taken from Zeleny (1982). Problems 5–7 were randomly generated with fully dense constraint and objective matrices. For Problems 8–10, the constraint matrices were fully dense, while the objective matrices were generated differently. Problems 11–15 were obtained from the interactive MOLP explorer (iMOLPe) of Alves et al. (2015). Problems 16–19 were taken from Steuer (1986), and Problem 20 was obtained from the Multi-Objective Problem Library (MOPLIB) of Lohne and Schenker (2015).

For the computational experiments, a MATLAB implementation of MOPPA based on Fraga and Amusat (2016) was employed, while BOA was implemented using BENSOLVE 2.0, the MATLAB-based implementation developed by Lohne and Weibing (2015). The two algorithms were applied to the same collection of MOLP instances to facilitate a direct comparison of their outputs. In Table 1, *n* denotes the number of decision variables, *m* denotes the number of constraints, and *q* represents the number of objective functions.

The comparison is based on the nondominated points obtained from each algorithm. Because BOA is an objective-space exact approach designed to determine the nondominated set, whereas MOPPA is a stochastic population-based approximation method, differences in the quality of the resulting nondominated points are expected to provide useful evidence concerning the relative performance of the two approaches. The results are presented in Table 1.

Table 1: Numerical results for individual problem

	Algorithms				BOA	MOPPA
Origin	**Prob.**	**N**	**M**	**q**	**Nondominated Points**	**Nondominated Points**
Ehrgott 2006	1	3	3	3	f1 = -2.00 f2 = 10.00 f3 = -5.00	f1 = -0.70 f2 = 7.12 f3 = -3.56
Zeleny 1982	2	2	2	2	f1 = -25000 f2 = -66000	f1 = -24880 f2 = -37320
"	3	2	4	3	f1 = -3.00 f2 = -7.50 f3 = -9.00	f1 = -3.10 f2 = -2.50 f3 = -4.40

"	4	7	4	3	f1 = -48.00 f2 = -32.00 f3 = -16.00	f1 = -13.90 f2 = -10.76 f3 = -2.65
Randomly generated with dense matrices	5	30	15	3	f1 = -236.52 f2 = -26.73 f3 = -137.31	f1 = -170.41 f2 = -189.70 f3 = -246.81
"	6	35	20	3	f1 = -363.82 f2 = -47.30 f3 = -136.71	f1 = -121.27 f2 = -41.10 f3 = -111.15
"	7	20	40	3	f1 = -108.70 f2 = -78.35 f3 = -17.77	f1 = -45.30 f2 = -40.20 f3 = -14.10
Randomly generated with dense matrices	8	15	45	3	f1 = -44.80 f2 = -23.60 f3 = -200.80	f1 = -24.40 f2 = -13.90 f3 = -170.80
"	9	30	50	3	f1 = -160.22 f2 = -92.15 f3 = -23.77	f1 = -120.94 f2 = -50.02 f3 = -21.56
"	10	30	60	3	f1 = 45.11 f2 = -24.58 f3 = -283.42	f1 = -30.15 f2 = -24.20 f3 = -163.31
iMOLPe	11	4	4	3	f1 = -40.00 f2 = -50.00 f3 = -10.00	f1 = -40.00 f2 = -50.00 f3 = -10.00
"	12	3	3	3	f1 = -2.00 f2 = -2.00 f3 = -4.00	f1 = -2.00 f2 = -1.92 f3 = -2.00
"	13	15	10	2	f1 = -363.82 f2 = -33.70	f1 = -143.05 f2 = -32.11
"	14	15	10	3	f1 = -343.50 f2 = -42.50 f3 = -158.75	f1 = -133.76 f2 = -44.05 f3 = -76.54
"	15	10	5	3	f1 = -226.40 f2 = -501.86 f3 = -351.14	f1 = -164.65 f2 = -47.33 f3 = -58.37
Steuer 1986	16	12	16	4	f1 = -5.56 f2 = -14.25 f3 = -8.25 f4 = -1.00	f1 = -5.76 f2 = -2.19 f3 = -3.05 f4 = -1.05
"	17	8	8	3	f1 = -14.48 f2 = -4.74 f3 = -6.93	f1 = -2.06 f2 = -4.68 f3 = -4.02

"	18	10	14	5	f1 = -4.90 f2 = -3.42 f3 = -4.38 f4 = -18.98 f5 = -9.27	f1 = -2.04 f2 = -1.42 f3 = -3.38 f4 = -8.98 f5 = -3.27
"	19	7	7	3	f1 = -3.38 f2 = -76.46 f3 = -49.57	f1 = -2.00 f2 = -30.01 f3 = -20.00
MOPLIB	20	100	20	3	f1 = -168.00 f2 = -124.00 f3 = -143.00	f1 = -121.80 f2 = -120.00 f3 = -126.03

4.2 Comparative Analysis of the Results

The results in Table 1 reveal a clear difference between the solutions generated by BOA and MOPPA. Overall, BOA produced nondominated points of higher quality for most of the test problems. This finding is consistent with the fundamental difference between the two approaches. BOA is an objective-space algorithm specifically designed to construct the nondominated set systematically, whereas MOPPA uses stochastic population-based search to approximate the nondominated frontier.

For Problem 1, BOA generated the point $(-2.00,10.00,-5.00)^T$, whereas MOPPA returned $(-0.70,7.12,-3.56)^T$. The BOA result therefore provides a stronger nondominated solution across the reported objective values. A similar pattern is observed for Problem 2, where BOA produced $(-25{,}000,-66{,}000)^T$, compared with $(-24{,}880,-37{,}320)^T$ from MOPPA. The difference is particularly pronounced for the second objective, indicating that MOPPA did not reach a point of comparable quality to the BOA solution.

The same general pattern is evident in Problems 4–10, which include randomly generated instances with relatively larger numbers of variables and constraints. For example, Problem 5 has 30 variables and 15 constraints, while Problems 9 and 10 each have 30 variables with 50 and 60 constraints, respectively. BOA generated the reported points with substantially more favourable objective values in these instances. This suggests that the objective-space approach retained an advantage even as the dimensions and complexity of the test problems increased.

The results for Problems 11–15, obtained from iMOLPe, provide additional evidence of the relative performance of the two methods. In Problem 11, both algorithms returned exactly the same nondominated point, namely $(-40.00,-50.00,-10.00)^T$. This represents an instance in which MOPPA successfully reproduced the point obtained by BOA. In Problem 12, however, the two solutions were slightly different, with BOA returning $(-2.00,-2.00,-4.00)^T$ and MOPPA returning $(-2.00,-1.92,-2.00)^T$. The difference becomes more pronounced in Problems 13–15, where BOA generally returned more favourable combinations of objective values than MOPPA.

The Steuer (1986) test problems, particularly Problems 16–19, further demonstrate the difference between the approaches. In these instances, which involve between three and five objectives, BOA consistently produced objective vectors that were generally superior to those obtained using MOPPA. The differences are particularly noticeable in Problems 18 and 19, where the objective values obtained by BOA are substantially different from those generated by MOPPA.

Finally, Problem 20, obtained from MOPLIB, represents the largest test instance in terms of the number of decision variables, with n = 100 and m = 20 constraints. BOA generated the nondominated point $(-168.00,-124.00,-143.00)^T$, whereas MOPPA returned $(-121.80,-120.00,-126.03)^T$. The BOA result is therefore superior for the reported objective vector, particularly for the first and third objectives.

4.3 General Performance of MOPPA and BOA

The results indicate that MOPPA is capable of generating feasible and competitive nondominated points across MOLP problems of different sizes and structures. Its ability to obtain meaningful solutions for relatively large instances demonstrates the usefulness of population-based stochastic search as an alternative to conventional exact methods. Nevertheless, the results also show that the quality of the points generated by MOPPA is generally below that obtained using BOA.

The superiority of BOA observed in Table 1 can be attributed to its objective-space construction procedure. Rather than relying on stochastic exploration of the decision space, BOA progressively approximates the attainable objective region and systematically identifies its nondominated boundary. This characteristic makes BOA particularly suitable when the primary objective is to obtain high-quality or complete representations of the nondominated frontier.

MOPPA, in contrast, relies on population initialization, fitness evaluation, selection, propagation, and elitist retention to search for promising solutions. Its stochastic nature provides flexibility and allows it to explore complex solution spaces without requiring exhaustive enumeration. However, because the method provides an approximation to the nondominated frontier, there is no guarantee that every run will identify the same nondominated points or reproduce the complete frontier obtained by an exact objective-space method.

It is important to note that the present comparison is based on the quality of the reported nondominated points rather than on computational time, number of nondominated points, convergence indicators, or statistical measures obtained from multiple independent MOPPA runs. Therefore, the results support the conclusion that BOA produced better-quality nondominated points for the selected instances, but they should not be interpreted as establishing that BOA is superior to MOPPA with respect to every possible performance criterion.

5.0 Conclusion

We have applied the Multi-objective Plant Propagation Algorithm of Fraga & Amusat (2016) and Benson's outer approximation algorithm of Benson (1998a) to multi-objective linear programming (MOLP). We have also reviewed the existing literature on these algorithms as well as compared the quality of solutions returned by BOA and MOPPA. Numerical results from a group of twenty (20) MOLP instances ranging from small to medium and realistic instances show that Benson's outer approximation algorithm of Benson (1998a) is superior to the Multi-objective Plant Propagation Algorithm of Fraga & Amusat (2016) in terms of the quality of nondominated points it returns in most of the problems considered.

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Author Contributions

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