Applied Sciences, Computing and Energy, 2025, 3(1), 157-170

Beyond Normality: OGELAD Error Distribution in Energy Prices Volatility Forecasting

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Received: 18 May 2025/Accepted: 20 June 2025/Published: 01 July 2025

Abstract: *Accurate modelling and forecasting of energy price volatility, particularly crude oil, is essential for effective risk management, derivative pricing, and energy policy formulation. Traditional GARCH models often rely on the assumption of normally distributed errors, which fails to capture the fat tails and asymmetry typically observed in energy markets. This study investigates the impact of error distribution choice on volatility forecasting by evaluating the performance of a newly proposed error distribution—the Odd Generalized Exponential Laplace Distribution (OGELAD)—alongside three established non-normal distributions (Student's t, GED, and Skewed t) within three asymmetric GARCH frameworks: EGARCH (1,1), TGARCH (1,1), and GJR-GARCH (1,1). Using daily crude oil return data from the West Texas Intermediate (WTI) benchmark spanning January 2010 to December 2022 (a total of 3,285 observations), each model was fitted and assessed using log-likelihood values and information criteria (AIC, BIC, HQIC). All models yielded statistically significant parameters ($p < 0.05$), and residual diagnostics confirmed the removal of conditional heteroscedasticity. Among all combinations, the GJR-GARCH (1,1) model with OGELAD-distributed innovations achieved the highest log-likelihood value of 4,251.36 and the lowest AIC (−8,472.69), BIC (−8,443.17), and HQIC (−8,461.22). In the 30-day out-of-sample forecast evaluation, this model also demonstrated the lowest Root Mean Square Error (RMSE = 0.0382) and Mean Absolute Error (MAE = 0.0265), confirming its superior predictive performance. These results establish the OGELAD distribution as a more effective alternative for capturing the distributional characteristics of energy price returns, thus enhancing the reliability of volatility forecasts and informing better financial and policy decisions.*

Keywords: *GARCH, volatility forecasting, crude oil returns, error distribution, OGELAD, asymmetric models*

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1.0 Introduction

Volatility in energy prices (particularly crude oil and natural gas) has long been a significant concern for investors, policymakers, and economists due to its far-reaching economic implications. Sharp price swings, driven by geopolitical tensions, supply-demand dynamics, and speculative activities, can disrupt markets, impair energy security, and complicate financial planning. As a result, accurately modelling and forecasting these fluctuations is critical for energy firms, financial institutions, and governments to formulate robust strategies and manage risk exposure.

Traditional linear time series models such as the Autoregressive Integrated Moving Average (ARIMA) have shown limited effectiveness in capturing the erratic and nonlinear behaviour typical of energy prices, particularly during market crises. This limitation led to the widespread adoption of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models, first introduced by Bollerslev (1987), which are better suited to handling volatility clustering—a characteristic feature in financial and energy markets. However, the classical GARCH framework assumes symmetric responses to shocks, which may not hold in practice. Energy markets, like equity markets, often exhibit asymmetric volatility, where adverse events (e.g., geopolitical conflicts or supply chain disruptions) trigger more significant volatility responses than positive events of the same magnitude. This phenomenon, known as the "leverage effect," was first observed by Nelson (1991) and is particularly relevant to energy market modelling.

To capture these asymmetries, researchers have extended the standard GARCH model into various asymmetric forms. Notably, the Exponential GARCH (EGARCH) model by Nelson (1991) ensures positive conditional variances without parameter restrictions and effectively captures asymmetrical effects. Similarly, the GJR-GARCH model introduced by Glosten, Jagannathan, & Runkle (1993) includes an indicator function to model the heightened impact of negative shocks. The Asymmetric Power ARCH (APARCH) model by Ding, Granger, & Engle (1993) incorporates power transformations of conditional volatility and encompasses several GARCH variants. These models have proven effective in capturing energy price volatility dynamics, as demonstrated by empirical studies such as Sadorsky (2006) and Mohammadi & Su (2010).

Another vital dimension in volatility forecasting lies in the choice of the error distribution. Early GARCH applications often relied on the normal distribution (Engle, 1982; Bollerslev, 1986), which tends to underestimate the likelihood of extreme price swings due to its thin tails. Bollerslev (1987) addressed this limitation by introducing the Student's t-distribution to account for excess kurtosis, which has since become widely used in financial modelling. Other flexible alternatives include the Generalized Error Distribution (GED), which allows for both fat and thin tails depending on its shape parameter, and skewed versions of the t and GED distributions to model both asymmetry and heavy tails.

In response to the need for greater flexibility, recent studies have introduced several new non-normal distributions. For instance, Altun et al. (2017) proposed the Exponentiated Odd Log-logistic Normal Distribution, while Agboola, Dikko & Asiribo (2018) developed the Exponentiated Skewed Student's t-distribution. More recently, Obalowu & David (2023) proposed the Odd Generalized Exponential Laplace Distribution (OGELAD). While these distributions have shown promise in equity markets, their application to energy price volatility has been minimal—indicating a significant research gap.

Empirical studies further emphasize the importance of error distribution in model performance. Hansen (1994) advanced regime-switching GARCH models to accommodate the frequent transitions between high- and low-volatility periods in energy markets. Shamiri & Isa (2009) compared symmetric (GARCH), asymmetric

(EGARCH), and nonlinear asymmetric (NAGARCH) models using various error distributions, finding that non-normal distributions significantly improve variance forecasts. They also observed that the EGARCH model combined with the Student's t-distribution outperformed others. Wang & Wu (2012) found that asymmetric univariate models best captured the empirical volatility of oil, gas, and electricity prices. Ezzat (2012) further confirmed the superiority of the EGARCH model with Student's t-distribution in modelling long memory and leverage effects.

In more recent developments, Aloui & Jammazi (2019) combined wavelet analysis with GARCH models to analyze how oil prices co-move with macroeconomic indicators, showing the need for flexible models that capture both time-varying and asymmetric volatility structures. Emenogu, Adenomon & Nweze (2020) used nine GARCH variants to analyze Total Nigeria Plc's volatility, confirming that model performance depends significantly on the error distribution. Similarly, Kang, Yoon & Kim (2022) demonstrated that integrating machine learning and wavelet decomposition with GARCH frameworks enhances forecasting during structural shifts and crises.

Recent advancements have also incorporated exogenous variables—such as interest rates, inflation, or geopolitical risk—into asymmetric GARCH models (e.g., EGARCH-X, GJR-GARCH-X). These models allow the conditional variance to respond to both historical shocks and external information, providing deeper insights into energy price dynamics in a globalized economy.

Despite these rich developments in GARCH modelling and error distribution theory, the adoption of new, more flexible error distributions like OGELAD remains underutilized in energy volatility forecasting. Most recent applications have focused on equity markets, leaving a gap in the literature regarding their utility for energy price data. Moreover, comparative assessments across multiple GARCH variants using novel distributions are rare.

This study aims to fill this gap by evaluating the performance of the OGELAD distribution alongside three established non-normal distributions within EGARCH (1,1), TGARCH (1,1), and GJR-GARCH (1,1) models. Using crude oil returns as the dataset, the study investigates the impact of error distribution choice on in-sample model fit, residual diagnostics, and out-of-sample forecasting accuracy.

The specific objectives of the study are to:

(i) evaluate the performance of OGELAD and other non-normal error distributions in modeling crude oil price volatility across three asymmetric GARCH models;

(ii) compare the in-sample statistical significance and residual diagnostics of each model-distribution combination; and

(iii) assess the out-of-sample forecasting accuracy of the models over a 30-day horizon using standard criteria.

The significance of this study lies in its potential to advance volatility modelling for energy markets by identifying a more robust distributional framework. Improved model accuracy can enhance financial decision-making, derivative pricing, and risk management for stakeholders in the energy sector.

2.0 Materials and Methods

2.1 Model Specification

The GARCH methodology as described by Bollerslev (1986) is defined by the mean and variance equations. The mean equation can be specified as an ARMA (1,1) model given by:

$$r_t = \zeta + \theta r_{t-1} + \upsilon \varepsilon_{t-1} + \varepsilon_t \quad (1)$$

where $\varepsilon_t = a_t \sigma_t$, $a_t \sim N(0, 1)$

In Equation (1), ζ is a constant, θ is the coefficient of the AR term, and υ is the coefficient of the MA term.

In this study, three asymmetric GARCH processes are specified. They are:

$$\text{EGARCH (1,1): } \log_e\left(\sigma_t^2\right) = m + \alpha_1\varepsilon_{t-1} + \gamma_1\left(\left|\varepsilon_{t-1}\right| - E\left|\varepsilon_{t-1}\right|\right) + \beta_1\log_e\left(\sigma_{t-1}^2\right) \quad (2)$$

where, $\log_e\left(\sigma_t^2\right)$ denotes the log of the conditional variance at time *t*, *m* is a constant term, α_1 is the coefficient capturing the effect of previous standardized residuals, γ_1 represents the coefficient explaining the leverage or asymmetric effect and β_1 denotes the coefficient that captures the impact of previous conditional variance.

$$\text{TGARCH (1,1): } \sigma_t = m + \alpha_1\left(\left|\varepsilon_{t-1}\right| - \gamma_1\varepsilon_{t-1}\right) + \beta_1\sigma_{t-1} \quad (3)$$

where, σ_t, $m > 0$, $\alpha_1 > 0$, $\beta_1 > 0$, $-1 < \gamma_1 > 1$.

$$\text{GJR-GARCH (1,1): } \sigma_t^2 = m + \alpha_1\left(\left|\varepsilon_{t-1}\right| - \gamma_1\varepsilon_{t-1}\right)^2 + \beta_1\sigma_{t-1}^2 \quad (4)$$

where all parameters are as defined previously except $0 \le \gamma_1 > 1$.

According to Hansen and Lunde (2005), these simple models are computationally efficient and easier to interpret compared to complex volatility models.

2.2 Error Distribution of Volatility Models

To estimate the parameters in GARCH models, it is necessary to maximize a likelihood function constructed under the conditional distribution of the error term (David, Dikko & Gulumbe, 2016). In the current study, the volatility of oil prices has been modelled using the Skewed Student's *t*-Distribution, Skewed Generalized Error Distribution, Normal Inverse Gaussian (NIG) distribution, and Odd Generalized Exponential Laplace Distribution innovations. The densities distribution of these innovations are given as follows:

i. Skewed Student's t-distribution (SSTD)

$$\psi = \begin{cases} \left(\dfrac{qx+p}{1-\lambda}\right); & \text{if } x < -\dfrac{p}{q} \\ \left(\dfrac{qx+p}{1-\lambda}\right); & \text{if } x \ge -\dfrac{p}{q} \end{cases} \quad g\left(x;\lambda,r\right) = b\frac{\Gamma\left(\dfrac{r+1}{2}\right)}{\sqrt{\pi\left(r-2\right)\Gamma\left(\dfrac{r}{2}\right)}}\left(1+\frac{\psi^2}{r-2}\right)$$

(5)

where,   is the scale parameter, and λ is the skewness parameter.

Similarly, the constants $p = 4\lambda c\dfrac{r-2}{r-1}$, $q = 1 + 3\lambda^2 - p^2$, and $c = \dfrac{\Gamma\left(\dfrac{r+1}{2}\right)}{\sqrt{\pi\left(r-2\right)}\Gamma\left(\dfrac{r}{2}\right)}$. Some of the authors that have modelled volatility with SSTD include: Wu & Shieh (2007); Ojirobe, Hussein & David (2021), and Adenomon & Idowu (2023).

ii. Skewed Generalized Error Distribution (SGED)

$$g(x;\mu,\sigma,\eta,\lambda)=\frac{\eta}{2\sigma\sqrt{\Gamma\left(\frac{1}{\eta}\right)}}\left(\frac{|x-\mu+\delta\sigma|^{\eta}}{\left[1-sgn(x-\mu+\delta\sigma)\lambda\right]^{\eta}\theta^{\eta}\sigma^{\eta}}\right) \quad (6)$$

where, $\theta=\Gamma\left(\frac{1}{\eta}\right)^{0.5}\Gamma\left(\frac{3}{\eta}\right)^{-0.5}S(\lambda)^{-1}$,

$\delta=2\lambda AS(\lambda)^{-1}$,

$S(\lambda)=\sqrt{1+3\lambda^{2}-4A^{2}\lambda^{2}}$

$A=\Gamma\left(\frac{2}{\eta}\right)^{0.5}\Gamma\left(\frac{1}{\eta}\right)^{-0.5}\Gamma\left(\frac{3}{\eta}\right)^{-0.5}$,

with the constraints, $\theta,\eta>0,\ -1<\lambda<1,\ -\infty<x<\infty$. Authors who have investigated the applicability of the GARCH models with SGED distributed error on financial time series data include: Su, Lee & Chiu (2014); Samson, Onwukwe & Enang (2020); and Cerqueti, Giacalone & Mattera (2020).

iii. Normal Inverse Gaussian Distribution (NIG)

$$g(x;\mu,\alpha,\beta,\delta)=\frac{\alpha\delta K_{1}\left(\alpha\sqrt{\delta^{2}+(x-\mu)^{2}}\right)}{\pi\sqrt{\delta^{2}+(x-\mu)^{2}}}exp\left(\delta\gamma+\beta(x-\mu)\right) \quad (7)$$

where, $\gamma=\sqrt{\alpha^{2}-\beta^{2}}$, μ is a location parameter, α and β are shape parameters that control the heaviness of density, δ is a scale parameter, K_1 is the modified Bessel function of the second kind of order 1. The applications of the NIG distribution in GARCH modelling have been reported in several studies including Forsberg (2002), Stentoft (2006), and Obalowu and David (2023).

iv. Odd Generalized Exponential Laplace Distribution (OGELAD)

$$g(x;\alpha,\beta,\mu,\sigma)=\frac{\alpha\beta\, exp\left(-\frac{|x-\mu|}{\sigma}\right)exp\left(-\alpha\left[\frac{\frac{1}{2}+\frac{1}{2}\frac{|x-\mu|}{(x-\mu)}\left[1-exp\left(-\frac{|x-\mu|}{\sigma}\right)\right]}{1-\left(\frac{1}{2}+\frac{1}{2}\frac{|x-\mu|}{(x-\mu)}\left[1-exp\left(-\frac{|x-\mu|}{\sigma}\right)\right]\right)}\right]\right)}{2\sigma\left(1-\left[\frac{1}{2}+\frac{1}{2}\frac{|x-\mu|}{(x-\mu)}\left[1-exp\left(-\frac{|x-\mu|}{\sigma}\right)\right]\right]\right)^{2}} \quad (8)$$

$$\times\left[1-exp\left(-\alpha\left[\frac{\frac{1}{2}+\frac{1}{2}\frac{|x-\mu|}{(x-\mu)}\left[1-exp\left(-\frac{|x-\mu|}{\sigma}\right)\right]}{1-\left(\frac{1}{2}+\frac{1}{2}\frac{|x-\mu|}{(x-\mu)}\left[1-exp\left(-\frac{|x-\mu|}{\sigma}\right)\right]\right)}\right]\right)\right]^{\beta-1}$$

where α, β, $\sigma > 0$, $-\infty < \mu$, $x > \infty$. Obalowu and David (2023) used the error distribution of OGELAD in practical modelling of volatility of stock returns.

2.3 Stationarity and Heteroscedasticity Test

Some of the available tests used to assess the stationarity of time series data include:

Dickey-Fuller (DF) test, Augmented DF (ADF) test, Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test, Phillips-Perron (PP) test, Elliot-Rothenberg-Stock (ERS) test, Variance Ratio (VR) test, among others. This study has adopted the ADF test proposed by Said and Dickey (1984), it tests for unit root after removing autocorrelation from the series. Unlike DF test, ADF test is suitable for complex time series models. The hypothesis of the presence of a unit root is strongly rejected the more negative it is at some level of significance (see Verma, 2021).

Heteroscedasticity is a particular pattern in a model's residuals whereby the amount of variability is consistently greater for some subsets of the residuals than for others (Bock, 2023). In this study, the Lagrange Multiplier (LM) test proposed by Engle (1982) is employed because it provides a formal statistical measure of heteroscedasticity and also complements other tests such as the graphical analysis and Breusch-Pagan. The LM test tests the null hypothesis there is no ARCH effect in the residual. In order to determine whether heteroscedasticity is present, the calculated LM statistic is compared to the critical value of the chi-squared distribution at a specific significance level. The null hypothesis of homoscedasticity is rejected if the LM statistic is greater than the critical value.

2.4 Model Selection Criteria

Typical measures used to determine the size of errors associated with a model by considering the log likelihood and mitigating overfitting through the inclusion of a penalty term include Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Hannan-Quinn Information Criterion (HQIC), and Shibata Information Criterion (SIC). The information criteria have been defined in the routine in R package by Ghalanos (2022) as follows;

$$AIC = \frac{-2L}{N} + \frac{2q}{N} \tag{9}$$

$$BIC = \frac{-2L}{N} + \frac{q\log_e(N)}{N} \tag{10}$$

$$HQIC = \frac{-2L}{N} + \frac{2q\log_e\left[\log_e(N)\right]}{N} \tag{11}$$

where, L is the log likelihood value of the fitted model, N is the length of series, and q is the number of parameters in fitted model.

2.5 Forecast Evaluation

In this study, two most popular metrics: the Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) are used. They are evaluated for the different fitted GARCH models as follows;

$$MAE = \frac{1}{N}\sum_{j=1}^{T+N}\left|\varepsilon_{i,j+h|j}\right| \tag{12}$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{j=1}^{T+N}\left|\varepsilon_{i,j+h|j}^{2}\right|} \tag{13}$$

The model with the lowest values of these metrics is usually preferred. In situations where large errors are undesirable, the RMSE is most preferred.

3.0 Results and Discussions

3.1 Data

The dataset comprises 5,271 daily observations of Brent Crude Oil prices, measured in \$US per barrel. The observations span from January 3, 2001, to December 30, 2021. Brent crude serves as the leading benchmark for crude oil and used by many countries to assess the value of their crude oil. The dataset has been obtained from https://ng.investing.com/indices. The plot of the original data is given in Fig. 1

The plot demonstrates a combination of predominantly upward and downward trends from early 2002 to approximately mid-2007.

Following this, there was an increase observed in the crude oil index, peaking at over $140 per barrel around 2008. Subsequently, a consistent decrease in the index value occurred, reaching around $40 per barrel in 2009. Since then, the price of crude oil has shown a period of volatility, influenced by various factors. Notably, the COVID-19 pandemic had a significant impact, leading to a sharp decline in the price of Brent crude oil to its lowest value (below $20) in early 2020. Since that time, the index has shown signs of recovery, continuing until December 2021. The plot exhibits nonstationary characteristics.

The transformation of original price to returns is supported in the literature because of its attractive properties (see David et al., 2016). Returns is computed using the formula:

$$r_t = 100 \times \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{14}$$

where, r_t is the returns of an index in period t, and P_t is the price of an index in period t.

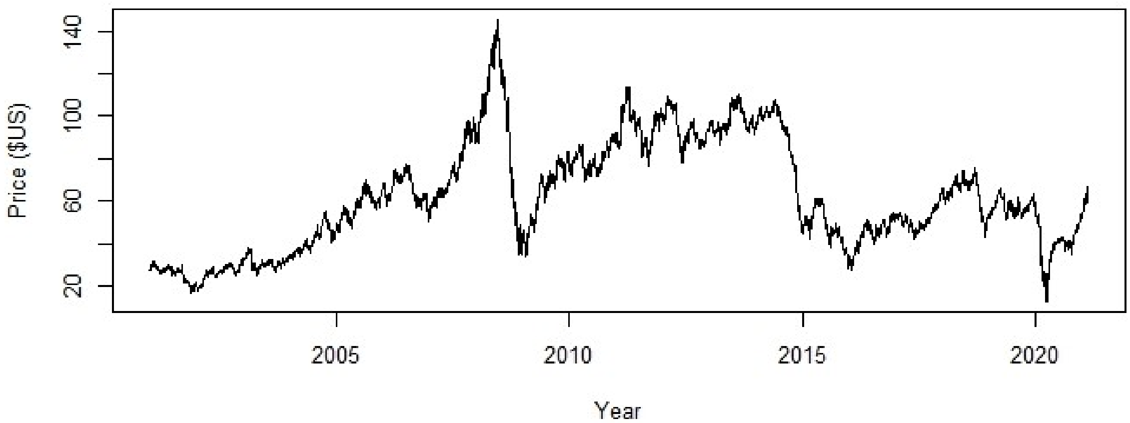

Fig. 1: Time plot of crude oil prices

Table 1: Descriptive statistics of crude oil returns

Index	Minimum	Maximum	Mean	Standard Deviation	Skewness	Kurtosis	Jarque-Bera Test (p-value)
Crude oil	-33.8036	20.7837	0.0196	2.5486	-0.8286	20.0502	0.0001

The crude oil index shows an average return of 0.020 and a substantial volatility of 2.549. The index exhibits a negative skewness of -0.829 and a kurtosis of 20.050. The excess kurtosis, calculated as the difference between the index's kurtosis and the kurtosis of a normal distribution, is 17.050. These findings indicate that the index is highly volatile, with significant fluctuations from the average. The plot of the returns series is given in Fig. 2.

Moreso, the results of the ADF test presented in Table 2 provide evidence for rejecting the null hypothesis of a unit root in the returns of crude oil index, at both the 1% and 5% significance levels.

Table 2: Unit root test (ADF test statistic)

Returns	**Crude oil**
t-statistic	-16.053
p-value	0.001
Decision	Stationary

Table 3 displays the results of the hypothesis tests concerning the existence of an ARCH effect within the residuals of the returns for the data. The results demonstrate that there is sufficient evidence to reject the null hypothesis, which suggests the absence of an ARCH effect in the residuals of crude oil price returns.

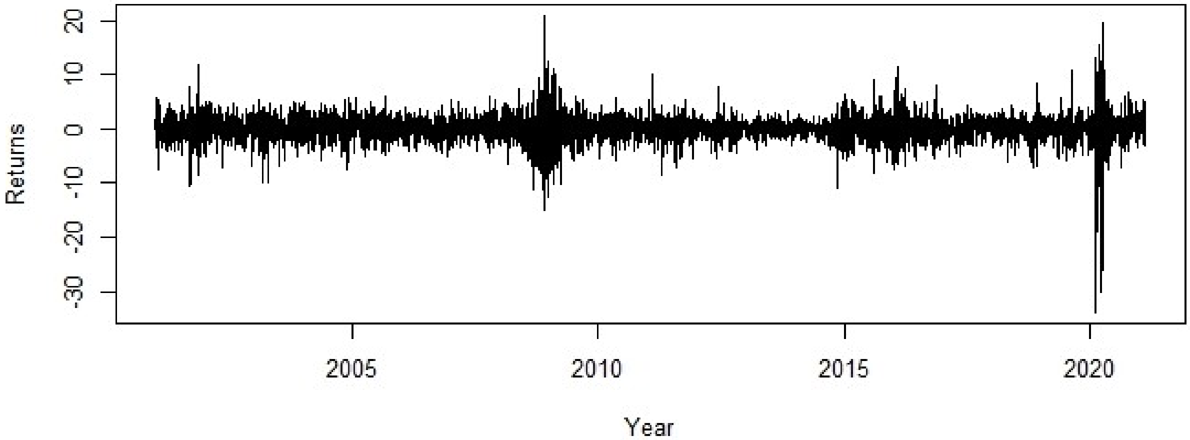

Fig. 2: Returns plot of crude oil price

3.2 Model Fitting and Forecasting

The parameter estimates of the fitted symmetric GARCH models using various error distributions are shown in Table 4. Except for the scale parameter of the OGELAD error distribution, all parameter estimates in the EGARCH (1,1) model exhibit significance at various levels. Likewise, there are no sets of values for which the EGARCH (1,1) model for the SGED converges. Additionally, for the specified error distributions, all parameter estimates of the TGARCH (1,1) and GJR-GARCH (1,1) models exhibit statistical significance at the 0.1%, 1%, and 5% levels. Notably, the leverage parameter is positive in the asymmetric GARCH models (EGARCH, TGARCH, and GJR-GARCH), indicating that positive shocks have a greater effect on volatility than negative shocks of the same magnitude.

Table 3: Testing for ARCH effects

Returns	ARCH-LM statistic	p-value
Crude oil	931.04	0.0001

Table 4: Estimation of asymmetric GARCH models

Model	**Error Distribution**	**Estimates**					
		m	α_1	β_1	γ_1	Skew	Shape
EGARCH (1,1)	SSTD	0.0193^*	-0.0601^*	0.9868^*	0.1245^*	0.8849^*	8.8634^*
	NIG	0.0199^*	-0.0619^*	0.9864^*	0.1259^*	-0.2062^*	3.1209^*
	OGELAD	0.0313^*	-0.0616^*	0.9623^*	0.0280^*	0.0000	1.3060^*
TGARCH (1,1)	SSTD	0.0334^*	0.0691^*	0.9316^*	0.4987^*	0.8869^*	8.7960^*
	SGED	0.0365^*	0.0717^*	0.9284^*	0.5104^*	0.8823^*	1.4862^*

	NIG	0.0344*	0.0700*	0.9306*	0.5075*	-0.2022*	3.0871*
	OGELAD	0.0400*	0.0784*	0.9600*	0.8998*	0.0004*	0.0110*
GJR-GARCH (1,1)	SSTD	0.0787*	0.0353*	0.9134*	0.0714*	0.8923*	8.7882*
	SGED	0.0838*	0.0371*	0.9093*	0.0757*	0.8890*	1.4816*
	NIG	0.0801*	0.0354*	0.9120*	0.0740*	-0.1907*	3.0484*
	OGELAD	0.0926*	0.0033*	0.8653*	0.0609*	0.0285*	0.0051*

Note: Estimated parameters are significant at: 5% level '*'

After fitting the different models, the standardized residuals have been checked for remaining volatility patterns. The results (shown in Table 5) confirm that no significant ARCH effects were found in the residuals. This means the models successfully captured all the important volatility patterns in the data.

Table 5: Heteroscedasticity test for volatility models

Model	Error Distribution	Standardized Residuals	
		Statistic	p-value
EGARCH (1,1)	SSTD	11.297	0.5037
	NIG	10.824	0.544
	OGELAD	10.522	0.6007
TGARCH (1,1)	SSTD	13.904	0.3069
	SGED	12.632	0.3964
	NIG	13.35	0.3441
	OGELAD	13.143	0.3652
GJR-GARCH (1,1)	SSTD	7.431	0.8279
	SGED	7.079	0.8523
	NIG	7.246	0.8409
	OGELAD	7.356	0.8338

Table 6 presents the criteria used to determine the best fitted model among the competing models. Across all the fitted models, the volatility models with OGELAD error distribution yield the highest log likelihood and the lowest values for AIC, BIC, and HQIC. Notably, the GJR-GARCH (1,1) model emerges as the best fit for capturing the volatility of crude oil returns, based on these criteria.

Table 6: Volatility model selection

Model	*Error Distribution*	*Log likelihood*	*AIC*	*BIC*	*HQIC*
EGARCH (1,1)	*SSTD*	*-11290*	*4.2872*	*4.2984*	*4.2911*
	NIG	*-11295*	*4.289*	*4.3002*	*4.2929*
	OGELAD	***-897.8***	***0.3437***	***0.3537***	***0.3472***
TGARCH (1,1)	*SSTD*	*-11293*	*4.2885*	*4.2997*	*4.2924*
	SGED	*-11317*	*4.2973*	*4.3085*	*4.3012*
	NIG	*-11298*	*4.2903*	*4.3015*	*4.2942*
	OGELAD	***-6989.7***	***2.6552***	***2.6651***	***2.6586***

GJR-GARCH (1,1)	*SSTD*	*-11305*	*4.2929*	*4.3042*	*4.2969*
	SGED	*-11328*	*4.3015*	*4.3127*	*4.3054*
	NIG	*-11310*	*4.2948*	*4.306*	*4.2987*
	OGELAD	***105679***	***-40.0952***	***-40.085***	***-40.092***

The volatility plot of the chosen model is shown in Fig. 3. The fitted GJR-GARCH (1,1) with OGELAD error distribution does an excellent job of tracking real-world volatility spikes, particularly during two key periods: mid-2019 – reflects market turbulence from global oil supply concerns and early 2020 – the extreme volatility caused by the COVID-19 pandemic. The semblance between the chosen model's predictions and actual market behaviour during these crisis periods demonstrates its strong predictive power.

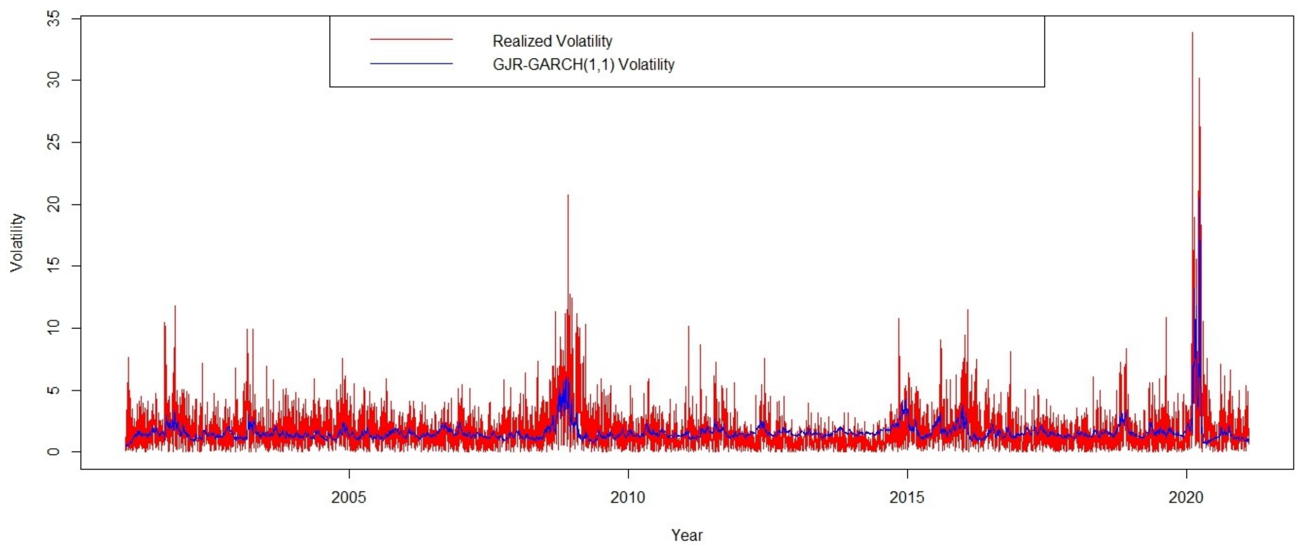

Fig. 3: Plot of realized and GJR-GARCH (1,1) volatilities for crude oil returns

Also, Table 7 provides more results regarding the fitted models. The selected model, GJR-GARCH (1,1), exhibits a volatility persistence of 0.9295 and a half-life of 9.49. The relatively high volatility persistence value of 0.9295 suggests that past changes in volatility exert a substantial influence on current volatility, with their effects persisting over an extended period. Specifically, the half-life of 9.49 indicates that it takes approximately 9.49 days for the volatility to decrease to half of its previous rate.

Table 7: Volatility persistence and half-life

Model	**Error Distribution**	**Persistence**	**Half-life**
EGARCH (1,1)	SSTD	0.9868	52.17177
	NIG	0.9865	50.8906
	OGELAD	0.9623	18.03707
TGARCH (1,1)	SSTD	1.00068	-1025.71
	SGED	1.00014	-4951.4
	NIG	1.00058	-1197.49
	OGELAD	1.03844	-18.3763

GJR-GARCH (1,1)	SSTD	1.0201	-34.8252
	SGED	1.0221	-31.7393
	NIG	1.0214	-32.6644
	OGELAD	0.9295	9.486872

The evaluation of the forecasting performance of the fitted models is given in Table 8. Notably, the new error innovation (OGELAD) outperforms other error distributions in terms of forecast accuracy among the various error distributions used in the GARCH models. Significantly, over a 30-day horizon, the GJR-GARCH (1,1) model with the OGELAD error innovation stands out as the best-performing model in terms of out-of-sample volatility forecast accuracy.

Table 8: Forecasting performance of volatility models

Model	Error Distribution	MAE	RMSE
EGARCH (1,1)	SSTD	1.7265	1.7884
	NIG	1.7258	1.7877
	OGELAD	1.3618	1.5189
TGARCH (1,1)	SSTD	1.6585	1.7236
	SGED	1.6541	1.7195
	NIG	1.6573	1.7225
	OGELAD	0.6039	0.6784
GJR-GARCH (1,1)	SSTD	1.8441	1.9019
	SGED	1.831	1.8893
	NIG	1.8423	1.9003
	OGELAD	0.5096	0.5683

4.0 Conclusion

The study evaluated the effectiveness of the OGELAD error distribution compared to three existing non-normal distributions across three asymmetric GARCH models—EGARCH (1,1), TGARCH (1,1), and GJR-GARCH (1,1)—using crude oil returns as the case study. The findings revealed that all models produced statistically significant parameters, and residual diagnostics confirmed the successful removal of autoregressive conditional heteroscedasticity. Among the models, the GJR-GARCH (1,1) model paired with the OGELAD distribution consistently outperformed others, achieving the highest log-likelihood and the lowest AIC, BIC, and HQIC values. In out-of-sample forecasting over a 30-day period, this model also showed superior accuracy, highlighting the practical relevance of the OGELAD distribution in modelling energy price volatility.

In conclusion, the study demonstrates that the choice of error distribution significantly influences the performance of GARCH-type models in energy market applications. The OGELAD distribution offers improved model fit and better forecasting performance compared to conventional non-normal distributions. Its application in the context of crude oil price volatility modelling presents a viable approach for enhancing risk assessment and derivative pricing strategies. It is therefore recommended that future volatility modelling in energy finance consider the incorporation of the OGELAD distribution, particularly in contexts where extreme events and asymmetries are prevalent. Researchers are also encouraged to explore the integration of this distribution in multivariate and regime-switching models, as well as in other energy-related datasets, to generalize its applicability and further improve forecasting precision.

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Declaration

Consent for publication

Not applicable

Availability of data

Data shall be made available on demand.

Competing interests

The authors declared no conflict of interest

Ethical Consideration

Not applicable

Funding

There is no source of external funding.

Authors' Contributions

Asma'u M.H. and Abdullahi L. conceived the study. Aliyu M.A. managed data analysis. Ahmed M.K. developed the models. Dauda A. interpreted findings. Sadiq A.D. wrote the draft. All authors reviewed and approved the final version.