Applied Science, Computing and Energy, 2026, 4(5), 659-676

# On The Application Of Multi-Objective Linear Programming To Niger Mills Company Plc Calabar, Cross River State, Nigeria

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**Received: 19 March 2026/Accepted: 14 July 2026/Published: 24 July 2026**

***Abstract*** *:This paper applies the multi-objective linear programming (MOLP) model to solve a real-world Niger Mills Company Plc Problem, Calabar, Cross River State, Nigeria for the first time. Several visits were made to the Company for the purpose of data collection. These data were used to identify and formulate the Niger Mills Company's problem as an MOLP problem. The problem was solved with our Matlab implementation of the extended multi-objective simplex algorithm (EMSA) of Nyiam & Salhi (2021a). A set of four (4) nondominated points (objective values) were returned by EMSA which are devoid of redundant ones. This result was further confirmed by the interactive multi-objective linear programming explorer (iMOLPe) of Alves et al. (2015) which implements the Vector Maximization Approach (VMA) of Steuer (1986) that also return the same set of nondominated points. Based on the result obtained, three different production plans were suggested for the Company's implementation while one nondominated point was recommended for deletion from being considered.*

***Keywords:*** *Multi-objective Linear Programming; Extended Multi-objective Simplex Algorithm; Interactive MOLP explorer; Nondominated points; Efficient solutions.*

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## 1.0 Introduction

Multi-Objective Linear Programming (MOLP) is a branch of Multiple Criteria Decision Making (MCDM) that seeks to optimize two or more linear objective functions subject to a set of linear constraints. MOLP has been a dynamic area of research over the years because of its relevance in practice; it has been widely applied in all areas of human endeavours and has become a useful tool in decision making. It is important to first define the concept of Multiple Criteria Decision Making (MCDM). Numerous definitions exit, but most researchers and practitioners in the field accept the following general one; MCDM refers to solving of planning and decision problems involving multiple and general conflicting objectives. "Solving" here means that decision makers (DMs) will choose one best alternative or most preferred solution from a set of available ones, Nyiam (2019).

In practice, MOLP is typically solved by the DM, with the support of the analyst looking for a most preferred solution (best) in the feasible region. We note here that, optimizing all the objective functions simultaneously is not usually possible due to the conflicting nature of the objectives. Consequently, the concept of optimality as was used in single-objective linear programming is replaced with that of efficiency or pareto-optimality first introduced by the Italian Economist Vilfredo Pareto (1906) and states that; a feasible solution is a Pareto optimal or efficient or non-dominated or non-inferior if there is no other feasible solution that is equal or better with respect to all the objective function in the model. Therefore, solving MOLP is understood as finding either all the efficient or non-dominated points or a subset of them, or a most preferred point depending on the purpose for which it is needed, Nyiam (2019).

**Also available @ https://doi.org/10.5281/zenodo.21737514**

This paper applies the extended multi-objective simplex algorithm (EMSA) of Nyiam & Salhi (2021a) and the interactive multi-objective linear programming explorer (iMOLPe) of Alves *et al*. (2015) for the first time to solve the Niger Mills Company's problem, Calabar, Cross River State, Nigeria. To the best of our knowledge, no application of an MOLP solution technique to the Niger Mills Company's problem has been carried out before. We intend to fill this gap here. To achieve this, several visits were made to Niger Mills Company Plc Calabar, Cross River State, Nigeria to collect relevant information and data which were used to formulate the Niger Mills Company's problem as an MOLP problem, the solution to the problem was obtained using a Matlab implementation of the Extended Multi-objective Simplex Algorithm (EMSA) of Nyiam & Salhi (2021a).

We stated in Nyiam & Salhi (2021b) that it was noted in Dauer (1987); Dauer & Liu (1990); Benson (1998a, 1998b & 1998c), Shao & Ehrgott (2008a & 2008b), Ehrgott *et al*. (2011), Ehrgott *et al*. (2012) and Lohne *et al*. (2014) that in practice, the DM prefers to base his or her choice of a most preferred solution on the objective values (nondominated points) rather than on the efficient solutions. In this study, we shall be presenting the nondominated points (objective values) returned by the two solution methods rather than the efficient solutions.

This paper is organized as follows. Section 2 introduces the MOLP general problem and its notations. Section 3 present a review of the relevant literature. The extended multi-objective simplex algorithm (EMSA) is provided in Section 4. Section 5 presents the Niger Mills Company Plc Calabar. The production process is presented in Section 6. Problem identification and formulation is provided in Section 7. Section 8 presents the solution to the problem. Section 9 discusses the result and finally, a conclusion is presented in Section 10.

### *1.1 The General MOLP problem*

The general MOLP problem as stated in Nyiam (2019) consist of $q$ linear objective functions defined on $X$ as follows:

$$f_1(x) = c_{11}x_1 + c_{12}x_2 + \cdots + c_{1n}x_n \quad \vdots$$

$$f_2(x) = c_{21}x_1 + c_{22}x_2 + \cdots + c_{2n}x_{n,}$$

$$\vdots$$

$$f_q(x) = c_{q1}x_1 + c_{q2}x_2 + \cdots + c_{qn}x_{n,}$$

Or

$$f_i(x) = \sum_{j=1}^{n} c_{ij}\, x_{j,}\ i = 1, \dots, q,$$

which are to be optimized subject to a set of $m$ linear constraints:

$$g_1(x) = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$
$$g_2(x) = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$
$$\vdots$$
$$g_m(x) = a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_{m,}$$

Or

$$g_r(x) = \sum_{j=1}^{n} a_{rj}\, x_j \leq b_{r,} \quad r = 1, \dots, m.$$

These linear inequalities together with the non-negativity conditions $x_j \geq 0$ for all $j$ forms the feasible set of solutions $X$. To "optimize" means either "minimize" or "maximize" the functions $f_i(x)$ over a nonempty convex polyhedron $X$ which amounts to finding a vector $\hat{x} \in X$ such that

$\sum_{j=1}^{n} c_{ij}\, x_j \leq \sum_{j=1}^{n} c_{ij}\, \hat{x}_j,\ or\ \sum_{j=1}^{n} c_{ij}\, x_j \geq \sum_{j=1}^{n} c_{ij}\, \hat{x}_j$ with $\sum_{j=1}^{n} c_{ij}\, x_j \neq \sum_{j=1}^{n} c_{ij}\, \hat{x}_j.$

Coefficients $c_{ij}$ can be positive, negative, or zero; they indicate the amount of gain (or loss) to be realized with respect to the $i^{th}$ objective per each unit of increase in the $j^{th}$variable. Coefficients $a_{rj}$ indicate how much of the $r^{th}$ resource must be expanded per each unit of increase in $x_j$. Nyiam (2019).

The above general MOLP problem can be written in compact form as

$$\text{`Max' or `Min'} \quad \begin{aligned} {c_1}^T x &= f_1 \\ {c_2}^T x &= f_2 \\ &\vdots \\ {c_q}^T x &= f_q \end{aligned} \tag{1}$$

Subject to $x \in X = \{x \in \mathbb{R}^n : Ax \leq b, b \in \mathbb{R}^m, x \geq 0\}$.

Or alternatively, as a linear vector optimization problem in matrix form as

$$\begin{aligned} \text{`Max' or `Min'} \quad & Cx \\ \text{Subject to} \quad & Ax = b \\ & X \geq 0 \end{aligned} \tag{2}$$

where $C$ is a $q \times n$ criterion matrix consisting of the rows $c_{k,}$ $k = 1, \dots, q$; A is an $m \times n$ constraint matrix, $b \in \mathbb{R}^m$ is the right hand side vector and "min or max" amount to finding an element $\hat{x} \in X$ such that the solution $\hat{x}$ is not worst that any other solution but in no way the best, that is, the point $C\hat{x}$ cannot be smaller than or equal to all point $Cx, x \in X$, Nyiam & Salhi (2021a).The feasible set in the decision space is $x \in X = \{x \in \mathbb{R}^n : Ax \leq b, b \in \mathbb{R}^m, x \geq 0\}$ and in the objective space it is *Y*= *Cx :* x ∈ *X*. The set *Y* is also referred to as the image of *X*. The nondominated faces in the objective space of the problem constitute the nondominated frontier while the efficient faces in the decision space of the problem constitute the efficient frontier, Nyiam & Salhi (2023).

A nondominated point in the objective space is the image of an efficient solution in the decision space, and the set of all nondominated points forms the nondominated set, Ehrgott *et al*. (2011).

An efficient solution of the problem is a solution that cannot improve any of the objective functions without deteriorating at least one of the other objectives.

Let $\hat{x} \in X$ be a feasible solution of (2) and let $\hat{y} = C\hat{x}$:

1. $\hat{x}$ is called efficient if there is no other $x \in X$ such that $Cx \leq C\hat{x}$ and $Cx \neq C\hat{x}$; correspondingly, $\hat{y} = C\hat{x}$ is called nondominated point.
2. $\hat{x}$ is called weakly efficient if there is no other $x \in X$ such that $Cx < C\hat{x}$; and $\hat{y} = C\hat{x}$ is called weakly nondominated point, Ehrgott (2006).

The set of all efficient solutions and the set of all weakly efficient solutions of (2) are denoted by $X_E$ and $X_{WE}$ respectively. $Y_N = \{Cx : x \in X_E\}$ and $Y_{WN} = \{Cx : x \in X_{WE}\}$ are the non-dominated and weakly non-dominated sets of (2) respectively, Benson (1998a).

## *2.0 Review of Related Literature*

Over the last few decades, there has been a growing awareness of the significance of multi-objective optimization in decision making processes; and in particular, MOLP has been an active area of research over the years and various solution approaches have been suggested to solve the problem.

We noted in Nyiam & Salhi (2021a) that Eiselt & Sandblom (2007) stated that, Evans & Steuer (1973), Philip (1972) and Zeleny (1974) all derived generalized versions of the simplex algorithm known as the multi-objective simplex algorithm (MSA) for computing all the efficient solutions of the problem. That due to Philip (1972), checks if an extreme point is efficient and subsequently determines if it is the only one present. If not, the algorithm computes them all. This MSA method however, may fail to perform at a degenerate vertex. This difficulty was overcome in a modified paper by Philip (1977).

The MSA of Evans & Steuer (1973) computes all the efficient solutions and unbounded efficient edges of the problem. This algorithm first verifies if the problem is feasible and has efficient solutions. Thereafter, it computes them all by moving from one efficient solution to adjacent efficient solution until all the efficient solutions have been found. An LP test problem is solved to determine the pivots that lead to efficient solutions.

The MSA of Zeleny (1974) also utilizes an LP test problem to generate the efficient solutions of the problem. The difference here is that, the vertices are tested for efficiency after they have been generated unlike in Evans & Steuer (1973) where the test problem determines pivots that lead to efficient vertices. Similarly, Yu & Zeleny (1974 & 1975) used the method in Zeleny (1974) to compute the set of all efficient solutions of the problem and formally presented a procedure for determining the efficiency of extreme points. It was noted that the decomposition of the parametric space into subsets associated with nondominated points can be obtained as a byproduct of applying the MSA in identifying the nondominated sets. The efficient solutions are derived from the efficient faces in a top-to-bottom search strategy. Numerical illustrations with three objectives were used to demonstrate the effectiveness of the method. In another paper, Yu (1976) applied their approach expanded in Yu (1975) to parametric LP. Two basic forms of the problem and two computational procedures for generating the efficient set were presented: the direct decomposition of the parametric space into subspaces associated with extreme points and the indirect algebraic method. From a numerical experience point of view, the indirect algebraic method outperforms the direct decomposition approach, Nyiam & Salhi (2021a).

Isermann (1977) presented a variant of the MSA of Evans & Steuer (1973) that solves fewer LPs when determining the entering variables. The algorithm first establishes whether an efficient solution for the problem exists, and solves a test problem to determine pivots leading to efficient vertices. The algorithm was implemented as a software called EFFACET in Isermann & Naujoks (1984).

The MSA of Gal (1977) computes the set of all efficient vertices and higher-dimensional faces. This method is meant to address the problem of determining efficient and higher dimensional faces that were not resolved in Evans & Steuer (1973) and Philip (1972). Here, efficient solutions are computed using a test problem and the algorithm also determines higher-dimensional efficient faces for degenerate problems which were only discussed in Isermann (1977) and Zeleny (1974) but not solved. The efficient faces are computed in a bottom-to-top search strategy unlike what was suggested in Yu (1974 & 1975).

Steuer (1986) presented the VMA and applied their MSA method in Evans & Steuer (1973) to parametric and non-parametric problems. Different methods for obtaining an initial efficient extreme point as well as different LP test problems were also presented. Efficient extreme points were generated through the decomposition of the weight space into finite subsets that provided optimal weights corresponding to extreme point solutions. Nyiam & Salhi (2021b).

Ehrgott (2006) applied the MSA of Evans & Steuer (1973) to solve problem instances with two and three objectives. Ecker & Kouada (1978) also presented a variant of the MSA of Evans & Steuer (1973). It was noted that algorithms normally start from an initial efficient solution and then, moved to an adjacent efficient one, following the solution of an LP test problem. Their proposed approach does not require the solution of any LP test problem to determine the efficiency of extreme points and the feasible region needs not be bounded. The algorithm generates all the efficient solutions and seems to have computational advantage over other known methods.

Ecker *et al*. (1980) introduced another variation of the MSA of Evans & Steuer (1973). This algorithm first generates the maximal efficient faces that are incident to a given efficient vertex (i.e. containing the efficient vertex) and ensures that previously computed efficient faces are not recomputed. This is done following a bottom-to-top search strategy as in Gal (1977), which dramatically improves computation time. The proposed method was illustrated with a degenerate problem in Yu (1975) in order to demonstrate its effectiveness. The proposed method was computationally more efficient than the approach in Yu (1975).

Similarly, the MSA variant of Armand & Malivert (1991) also generates the set of efficient solutions even for degenerate problems. The method follows a bottom-to-top search strategy and used a lexicographic selection rule to select the leaving variables during iteration, which proves effective when solving degenerate problems. The algorithm was successfully tested on a number of degenerate instances and was effective in solving them all.

Hussein (2024) proposed a novel rough interval MOLP (RIMOLP) model to solve problems under uncertainty. The model used rough set to represent uncertain coefficient as intervals with lower and upper bounds. The RIMOLP model was applied to a real-world problem at the Baghdad Water Department. The author transform the RIMOLP into an upper and lower bound interval multi-objective Linear programming (IMOLP) problem and solved using the weighted sum method. A numerical illustration was used to demonstrate the applicability of the method. It was concluded that a new approach to handle uncertainty in MOLP problems using rough set theory and its application in real-world has been introduced.

Nahar, *et al*. (2024) suggested a statistical averaging technique to solve Multi-Objective Linear Fractional Programming Problems (MOLFPPs) and MOLP problems. The technique employs arithmetic averaging and geometric averaging method as well as the simplex algorithm to find efficient solutions to the problem. This approach offers a new solution method for solving MOLP problems.

Nyiam & Salhi (2023) applied the Multi-objective Plant Propagation Algorithm (MOPPA) and Non-Dominated Sorting Genetic Algorithm II (NSGA-II), to solve MOLP problems for the first time. The computational results obtained were compared to four prominent exact methods: the extended multi-objective Simplex Algorithm (EMSA) of Nyiam & Salhi (2021a), affine scaling interior multi-objective linear programming (ASIMOLP) of Arbel (1993), Benson outer approximation (BOA) of Benson (1998a), and the parametric simplex algorithm (PSA) of Rudloff *et al*. (2015). Their findings indicate that MOPPA outperforms NSGA-II in terms of the quality of nondominated points it returns and compares favourably on large instances where exact methods failed; while NSGA-II provides a more evenly or uniformly distributed approximation of the nondominated frontier. It was also noted that, MOPPA and NSGA-II are viable alternatives to exact methods for solving realistic MOLP instances.

Matahari *et al*. (2023) applied an MOLP model to solve the cellular manufacturing systems problem. The cellular manufacturing systems problem was a three-objective problem with weighted completion time, transportation cost and machine idle time as objectives. The problem was solved using the Z-constraint method and NSGA-II for which non-dominated points were found. It was noted that, the results demonstrate the application of MOLP in optimizing cellular manufacturing systems and showcasing the power of MOLP techniques in solving real-world complex problems.

Valipour & Yashoobi (2022) presented a linearization method for solving MOLFP problems. This approach transforms the MOLFP into an MOLP and solved using the weighted sum method. Though it was noted that optimizing weighted sum objectives does not guarantee locating efficient solutions in a non-convex set, and propose modifications to enhance solution efficiency.

Mozaffari *et al*. (2022) proposed an MOLP model which incorporate ratio data into the production possibility set to solve Data Envelopment Analysis (DEA) problems. The Zion & Wallenius (1976) interactive method was used to solve the resulting MOLP problem for which efficient solutions were generated upon interaction with the DMs. The DMs sets targets and choose their preferred solution leading to the determination of the efficient solutions.

Nyiam & Salhi (2021a), extended the MSA of Evans & Steuer (1973) herein referred to as the so-called EMSA to generate all the nondominated points of the problem devoid of redundant ones. This extended variant was then compared with Benson's (1998a) outer approximation algorithm (BOA) that also computes all the nondominated points of the problem. The results show that the total number of nondominated points returned by EMSA is the same as that returned by BOA in most of the problems considered. In a similar paper, Nyiam & Salhi (2021b), presented yet another computational investigation of EMSA, ASIMOLP and BOA. The criteria for evaluation here was the quality of nondominated points returned by the three algorithms. The results of various numerical illustrations show that EMSA and BOA outperforms ASIMOLP in terms of the quality of nondominated points they returned, however, ASIMOLP excels in terms of computational efficiency.

Fashoto & Sani (2021) proposed an approach to solve MOLP with fuzzy objective functions. The approach was implemented in MATLAB and the results show the effectiveness and ease of application of the proposed method in solving benchmark problems. It was noted that the study contributes to the field of MOLP by providing a novel approach to handle fuzzy objective functions, which is essential in real-world applications where uncertainty and imprecision are inherent. Rivaz *et al*. (2016) introduced an MOLP algorithm to solve interval MOLP problems using the minimax criterion method. The algorithm identifies efficient solution of the problem so long as such solution exist. A numerical example was used to demonstrate its effectiveness and applicability.

Shao & Ehrgott (2015) presented a new algorithm for solving linear Multiplicative Programming Problems (MPPs), which is a class of global optimization problems using MOLP technique to efficiently compute the entire set of non-dominated points. The algorithm solves only one linear program in each iteration, thereby improving the computation time. Computational experiments demonstrate the effectiveness of the proposed method over existing global optimization methods. Doliente & Samsatil (2021) presented an MOLP model to optimize rice value chain for efficiency and sustainability. The model integrates rice, energy, fuel and chemicals production as well as minimize cost of production and greenhouse gas emissions. Numerical result show that streamlining value chains and integrating energy with rice production can increase productivity and reduce gas emission.

Ibisiki, *et al*. (2022) applied linear programming techniques to solve the wheat flour production problem in a flour mill industry. The study focuses on enhancing production efficiency and reducing costs by formulating a linear programming model that represent the operations of the company. This study provides valuable insights into the application of MOLP technique to various processes of the flour milling and grain production industries.

Capossio, *et al*. (2022) applied an MOLP model to find the ideal drying temperature required for transforming watermelon into flour for bakery products as well as developed a neutral model to predict the effects of drying temperature on fire processes indicators, acidity, pH, water holding capacity, oil-holding capacity and drying time. The MOLP procedure identified a temperature range of between 67.3ºC to 73.1ºC as the compromise nondominated temperature range for balancing the conflicting indicators, resulting in efficient value of each indicator.

It is worthy to note here that, of all the MSA variants discussed earlier, it was stated in Schechter & Steuer (2005) that, that of Evans & Steuer (1973) is the most popular and successful for generating all efficient solutions of the problem.

## 3.0 The Extended Multi-Objective Simplex Algorithm (EMSA)

A typical MSA is that of Evan & Steuer (1973) and can be found in Ehrgott (2006). Its extended version, (EMSA) of Nyiam & Salhi (2021a) is what is described here. We consider this algorithm because most of the MSA methods discussed earlier are either based on or are variants of the MSA of Evans & Steuer (1973). EMSA works in the decision space and generates the set of all efficient solutions as well as return all the nondominated points of the problem devoid of redundant ones.

We noted in Nyiam & Salhi (2021a) that in an MOLP problem, only one of the following situation can occur; the problem can be infeasible, meaning that the feasible set $X$ is empty ($X = \emptyset$); the problem may be feasible, that is ($X \neq \emptyset$) but may not have efficient solutions, that is $X_E = \emptyset$; or it is feasible and has efficient solutions $X_E \neq \emptyset$. This Algorithm handles this situation in three phases: In the first phase, it finds an initial basic feasible solution by solving an auxiliary LP or stop with the conclusion that ($X = \emptyset$); in the second phase, another auxiliary LP is solved to find an initial efficient basis $B$ or stop with the conclusion that $X_E = \emptyset$; while in the final phase, the algorithm pivots from an initial efficient basis to an adjacent efficient basis to determine all efficient solutions and also return the corresponding nondominated points of the problem devoid of repeated ones. Its implementation stores a list of efficient bases $L_1$ to be processed, a set $X_E$ of efficient solutions for output and a set $Y_N$ of nondominated points for output, see Nyiam & Salhi (2021a) for more details. The pseudo-code of extended MSA adapted from Nyiam & Salhi (2021a) is presented below:

**Algorithm 1 Extended Multi-Objective Simplex Algorithm (Nyiam & Salhi 2021a).**

**Input**: *A, b, C : (Data of MOLP Problem)*
**Initialize**: *Set* $L_1 \leftarrow \emptyset$, $L_2 \leftarrow \emptyset$, $X_E \leftarrow \emptyset$, $Y_N \leftarrow \emptyset$;
*Phase I: Solve the LP* $min\,\{e^T z : Ax + Iz = b, x, z \geq 0\}$. *If the optimal value of this LP is nonzero, STOP,* $X = \emptyset$;
*Otherwise* $x^0$ *is a basic feasible solution of MOLP*
*Phase II: Solve the LP* $min\,\{u^T b + w^T C x^0 : u^T A + w^T C \geq 0, w \geq e\}$. *If it is infeasible, STOP,* $X_E = \emptyset$. *Otherwise* $(\hat{u}, \hat{w})$ *is an optimal solution.*
*Find optimal basis B & basic feasible* $\bar{x}$ *of LP* $min\,\{\hat{w}^T Cx : Ax = b, x \geq 0\}$;
*Set* $L_1 \leftarrow \{B\}$, $L_2 \leftarrow \emptyset$, $X_E \leftarrow \{\bar{x}\}$, $Y_N \leftarrow \{C^T\bar{x}\}$;
**while** $L_1 \neq \emptyset$ **do**
*Choose* $B \in L_1, L_1 \leftarrow L_1 \backslash \{B\}$, $L_2 \leftarrow L_2 \cup \{B\}$;
*Compute* $\tilde{A}, \tilde{b}$, *and R according to B*;
$N_E \leftarrow N$;
**for** *all* $j \in N$
*Solve the LP max* $\{e^T v : Rz - r^j \sigma + Iv = 0; z, \sigma, v \geq 0\}$.
*If this LP is unbounded* $N_E \leftarrow N_E \backslash \{j\}$;
**for** *all* $j \in N_E$
**for** *all* $i \in B$
**if** $B' \leftarrow (B \backslash \{i\} \cup \{j\}$ *is feasible,* $B' \notin L_1 \cup L_2$, *let* $\bar{x}'$ *be it basic solution* **then;**
$L_1 \leftarrow L_1 \cup B'$;
$X_E \leftarrow X_E \cup \{\bar{x}'\}$;
$Y_N \leftarrow \{C^T \bar{x}'\}$;
**endif**;
**endfor;**

```
            endfor;
        endfor;
    endwhile
        Output: X_E: The set of Efficient Solutions
                Y_N: The set of Nondominated Points
```

# 4.0 The Niger Mills Company Plc Calabar, Cross River State, Nigeria

## 4.1 Incorporation & History

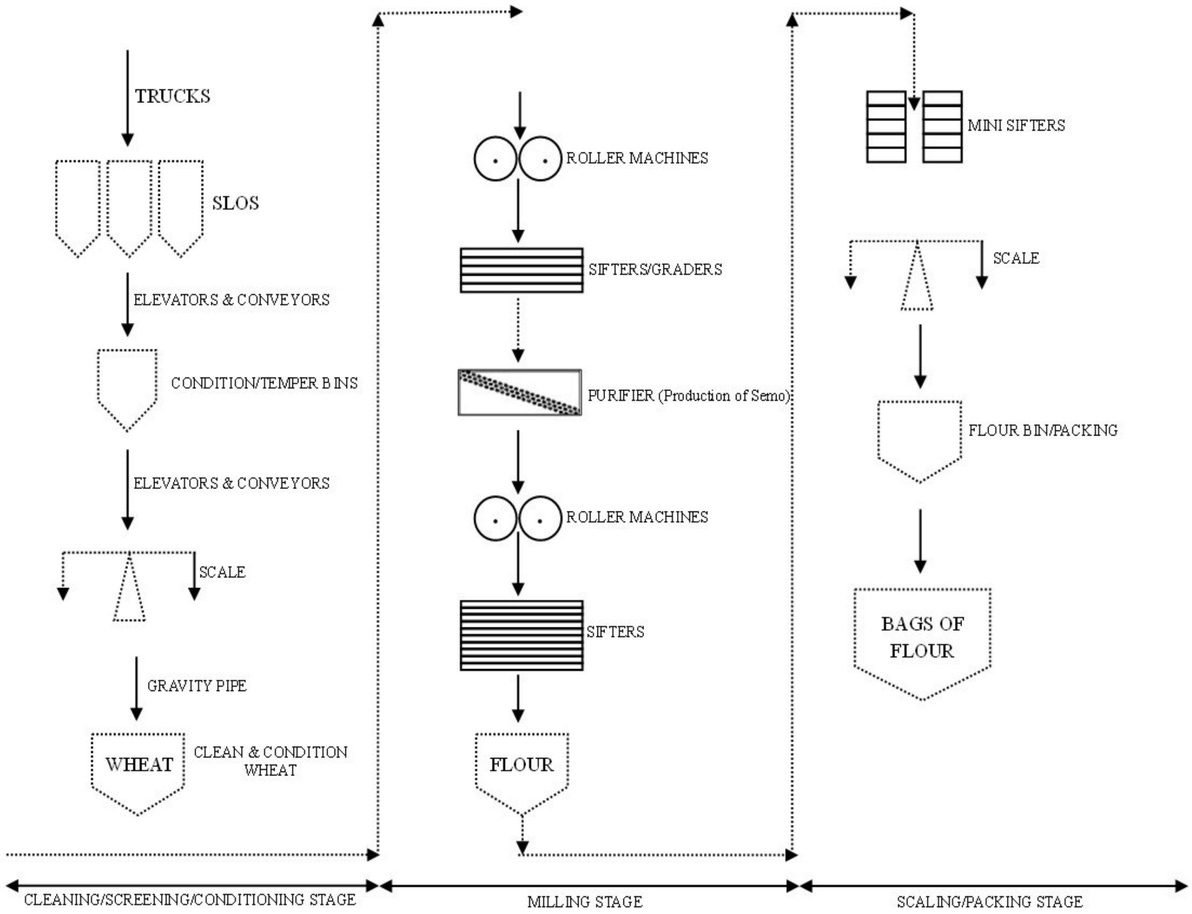


**Fig 1: Showing the Production Process**

## *4.2 The Screening, Cleaning and Conditioning Stage:*

This is the first stage of the production process; 18,000 tons of imported wheat monthly is conveyed with the use of trucks into three silos tanks (storage facilities). With the use of elevators and conveyors (which are a set of auxiliary machines) for cleaning the wheat, the wheat is being sorted from waste, stones, chaffs, foreign seeds, dirts and m*et al*lic objects etc. the wheat is then conditioned by controlled addition of water for storage in the temper bins, from the temper bins, the wheat is then transferred with the use of elevators/conveyors to a scale for scaling in tips of 50kg, and then through the gravity pipe to a wheat bin. Here, the wheat is cleaned, conditioned and ready for milling. We were made to understand by the chief Miller of the company that the total time taken for this stage is about an hour which translates to 60minutes.

## *4.3 The Milling Stage*

Here, 750tons of conditioned and cleaned wheat per day is passed into the roller machines for grinding to expose the endosperm for gradual flour extraction. The exposed endosperm is then moved into sifters/graders which are equipped with sieves of different apertures to separate the flour from other stock, and then to the purification of stock or middling for further reduction to flour. At this point, Semovita is produced. The purified stock is then passed into another set of roller machines for further grinding of pure coarse and fine middling into flour, which is then moved through the sifters for complementary sifting, and later to a flour bin for storage. The time taken to complete this stage is about 15minutes.

### *4.4 Scaling/Packing Stage*

This is the final stage of the production process. From the flour bin, the product (flour) is moved through mini sifters for a thorough and final sifting before bagging.

Then to a scale, to be scaled and sealed with tips of 50kg and later to the final surge flour bin for packing into bags. This stage takes only 5 minutes.

All things being equal, Niger Mills Company Plc Calabar operates for 24hrs in a day, and runs from Mondays through Saturdays (i.e. six working days in a week) and 24 days per month, this translates to 288 working days per annum. The company's management set aside Sundays for maintenance and general servicing of its plant for smooth and continuous operation.

As we have stated earlier, that Niger Mills Company Calabar produces four different products namely, Flour, Semovita, Whole Wheat and Wheat Offal (Animal Feed) from one source of raw material (Wheat). These four products are produced by passing sequentially through three different stages of production process viz, the Cleaning/Screening/Conditioning Stage, the Milling Stage and the Scaling/Packing stage. Figure 2 above shows the sequence of the production process and the time taken for each product to be produced.

From the above figure, 750 tons of wheat per day is poured into the condition/temper bins to be cleaned and conditioned before milling. This process usually takes 60 minutes. The same quantity is then passed into the milling stage for grinding. It takes additional 15 minutes for Semovita to be produced from the ground wheat. Further grinding of coarse and fine middling leads to the production of flour in an additional 5 minutes. It takes another 5 minutes for whole wheat and wheat offal to be produced respectively. Finally, it usually takes 5 and 3 minutes to scale and pack the products into 50kg and 10kg bags respectively.

## 5.0 Problem Identification/Formulation

From the description in Sections 6, one can deduce that the actual problem confronting Niger Mills Company Plc Calabar is that of allocation her scarce resources between the four competing activities (Flour, Semovita, Whole Wheat and Wheat Offals) in order to achieve maximum profit while minimizing the production cost. A company operating in this way, as described above, may likely have the following constraints:

i. Raw material availability
ii. Machine or Production time
iii. Personnel Cost
iv. Cost of Electricity
v. Utility Cost and
vi. Market or Demand Constraint etc.

Of all these constraints mentioned above, production time, raw material and the market constraints appears to be most predominant because they vary significantly as shown in appendix B & D.The cost of generating electricity, because the company does not depend on public power supply, utility and Personnel costs are termed "fixed costs" because they appear to be constant in the period under study, i.e. they do not vary significantly over a one-year period as shown in Fig. 3.

no role in the determination of objective values (nondominated points) given the Company's existing facilities and structure. In addition, our decision will not affect fixed costs.

The Company's only source of income is through the profit that is generated from the sale of its four products (Flour, Semovita, Whole Wheat and Wheat Offal). We let the decision variables of the problem be $x_1$ (i.e. the quantity to be produced of flour in 50kg bags), $x_2$ (the quantity to be produced of semovita in 10kg bags), and $x_3$ (the quantity to be produced of whole wheat also in 10kg bags) and $x_4$ (the quantity to be produced of wheat offal in 50kg bags) respectively. We also let $p_1$ be the profit per unit of flour, $p_2$ be the profit per unit of semovita $p_3$ be the profit per unit of whole wheat and $p_4$ be the profit per unit of wheat offal. Similarly, we also let $c_1$ be the cost per unit of flour. $c_2$ be the cost per unit of semovita. $c_3$ be the cost per unit of whole wheat and $c_4$ be the cost per unit of wheat offal.

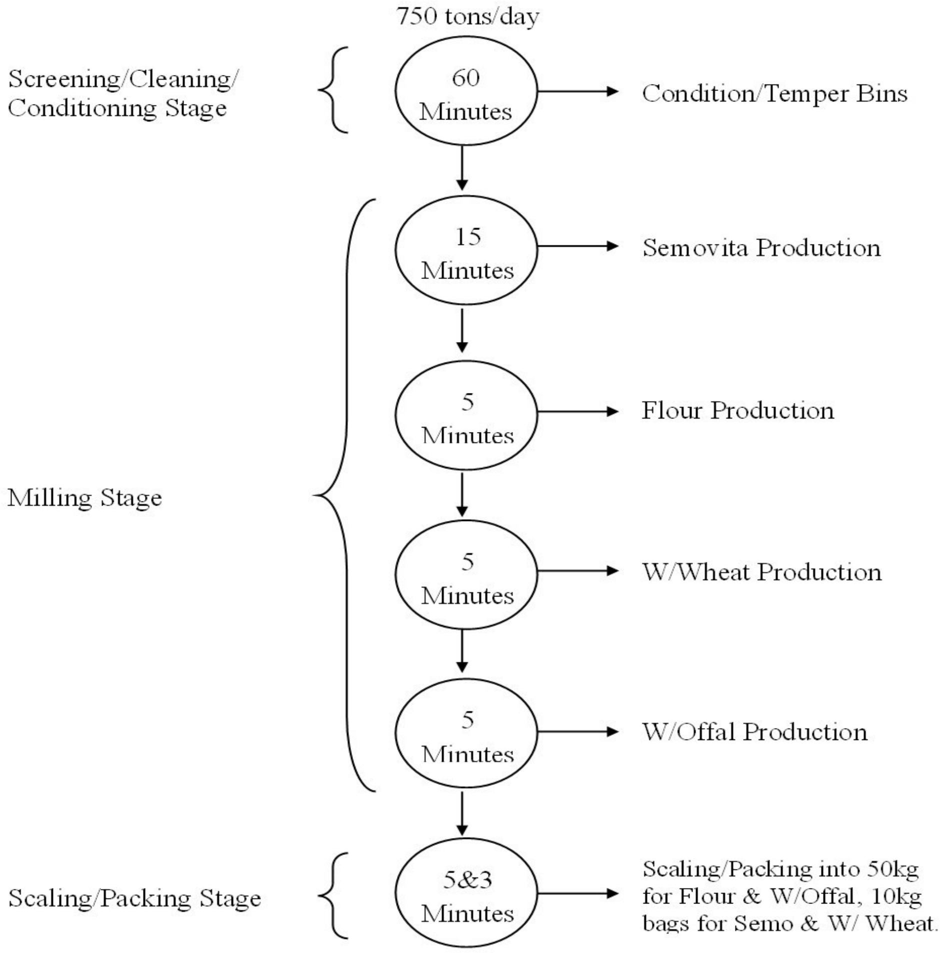


**Fig 2: The Sequence of Production Process**

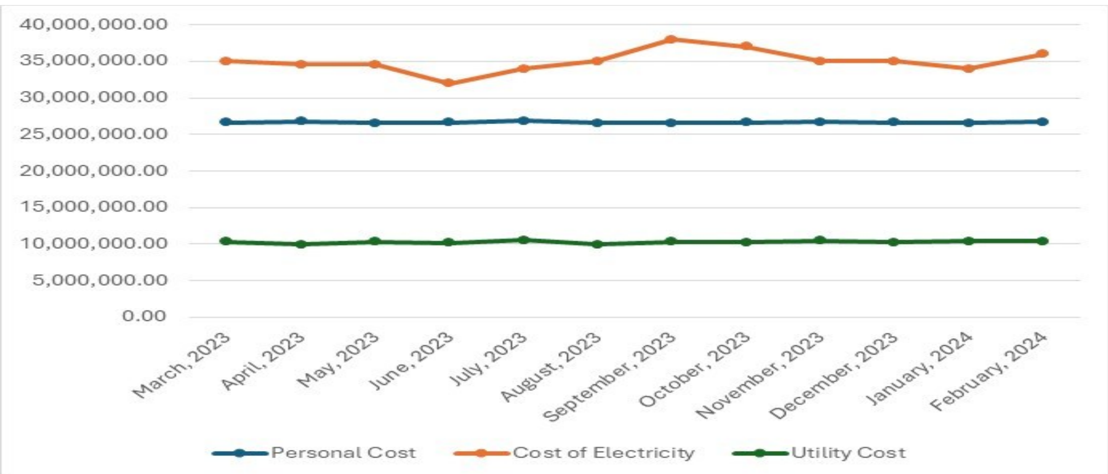


**Fig. 3: Graph of fixed costs**

It is worthy to note here that, fixed costs play The total profit ($f_1$) that would be made from the sales of these four products is a function of the decision variables and is related to the variables as follows:

Total profit equal to net revenue per unit of product times the number of products produced of the product, summed over all products. This can be expressed mathematically as

Total Profit ($f_1$) = $p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4$ (3)

while the total cost ($f_2$) is also related to the variables as follows:

Total Cost ($f_2$) = $c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4$ (4)

We seek to determine the different levels of these competing activities ($x_1$, $x_2$, $x_3$ and $x_4$) of the company so as to maximize ($f_1$) as well as minimize ($f_2$) subject to the functional constraints identified above as production time, raw material and the market (demand) constraints.

### *5.1 Problem Constraints*

From the explanations in the preceding section, three functional constraints were identified. These are the market or demand constraint, the raw material availability and the production or machine time constraint.

#### *5.1.1 Description of bound in the market constraint*

From the sales Department of the company, the maximum quantity of each product (i.e. maximum daily demand) that is sold per day is given as:

Flour is 8,500 (50kg) bags, Semovita is 1,234 (10kg) bags, Whole wheat is 1,244 (10kg) bags and Wheat offal is 2,800 (50kg) bags. Therefore, these quantities 8,500 (50kg) bags of flour, 1,234 (10kg) bags of semovita, 1,244 (10kg) bags of whole wheat and 2,800 (50kg) bags of wheat offal are the respective bounds of the market constraints and this is shown clearly in appendix D. Therefore, the mathematical statement of these demands for Flour, Semovita, Whole wheat and Wheat offal is

$$x_1 \leq 8{,}500 \quad (5)$$
$$x_2 \leq 1{,}234 \quad (6)$$
$$x_3 \leq 1{,}244 \quad (7)$$
$$x_4 \leq 2{,}800 \quad (8)$$

#### *5.1.2 Estimation of bound in the production time constraint*

To estimate the bound in time constraint, we recall that there are three different stages in the production process namely, screening/cleaning/conditioning stage, milling stage and the scaling/packing stage. We were also told in Section 6 that it took 60 minutes to screen, clean and condition the wheat before milling. At the milling stage, once the process is on, it normally takes 15 minutes for the first product (semovita) to be produced and 20 minutes for flour to be produced; 25 minutes for whole wheat to be produced and 30 minutes for wheat offal to be produced. Finally, in the scaling and packing stage, it took 5 minutes for flour and wheat offal to be scaled and packed into their respective 50kg bags and only 3 minutes for semovita and whole wheat to be scaled and packed into 10kg bags. The stages time allocated to the products is limited to 24hrs per day, which translates to 1440minutes. This implies that, the company can only run for 1,440 minutes in a day and this also imposes the restriction that:

$60x_1 + 60x_2 + 60x_3 + 60x_4 \leq 1{,}440$ (9);

We recall also that, at the milling stage, it took 15 minutes to produce semovita, 20 minutes to produce flour, 25 minutes for whole wheat and 30 minutes to produce wheat offal. This restriction is expressed mathematically by the inequality:

$20x_1 + 15x_2 + 25x_3 + 30x_4 \leq 1440$ (10);

The final stage of the production process is the scaling and packing stage. This takes 5 minutes for flour and wheat offal to be scaled and packed into their respective 50kg bags; while 3 minutes to scaled and packed semovita and whole wheat into their respective 10kg bags and leads to the restriction that:

$5x_1 + 3x_2 + 3x_3 + 5x_4 \leq 1440$ (11).

### *5.1.3 Description of bound in the raw material constraint*

It was also observed in Section 6 that 18,000 tons of raw material (wheat) was imported monthly by the company; out of which 750 tons is used daily for the production of flour, semovita, whole wheat and wheat offal. This means that, 750tons of wheat is the maximum daily requirement by the company and hence, it is the bound for raw material constraint. The first row in Appendix E implies that each unit of Flour produced would use 1/20 of the raw material (wheat), because one ton of wheat consumed, produces 20 (50kg) bags of flour, and a ton is equal to 1000kg, this also applies to wheat offal. While each unit of semovita produced would use 1/100 of wheat (raw material), because one ton of wheat consumed, produces 100 (10kg) bags of semovita and whole wheat, whereas only 750tons is available daily. This restriction is expressed mathematically by the inequality:

$$1/20x_1 + 1/100x_2 + 1/20x_3 + 1/100x_4 \leq 750 \quad (12)$$

### *5.2 Determination of Net Revenue (Profit)*

From the accounts department, it was learnt that the cost of producing a 50kg bag of flour ($c_1$) is ₦27,568.00; while that of producing a 10kg bag of semovita ($c_2$) is ₦9,786.00; that of producing a 10kg bag of whole wheat ($c_3$) is ₦4,048.00 and that of producing wheat offal ($c_4$) is put at ₦2,889.00. Also from the sales department, it was learnt that the selling price for a bag of flour is ₦42,301.00, the selling price for a bag of semovita is ₦11,000.00, that for selling a bag of whole wheat is ₦9,000.00 and that for selling a bag of wheat offal is ₦6,751.00. By subtracting the direct cost of each product from the selling price, we have the net revenue (profit) of ₦14,733.00, ₦1,214.00, ₦4,952.00 and ₦3,852.00 made on selling a bag of Flour, Semovita, Whole wheat and Wheat offal respectfully as shown in Appendix C.

Therefore, the profit per unit ($p_1$) of $x_1$ is ₦14,733.00; profit per unit ($p_2$) of $x_2$ is ₦1,214.00; while profit per unit of ($p_3$) of $x_3$ is ₦4,952.00 and profit per unit ($p_4$) of $x_4$ is ₦3,852.00. Substituting these values into eqns. (3) and (4) yields:

$$f_1 = 14,733x_1 + 1,214x_2 + 4,952x_3 + 3,852x_4 \quad (13)$$

$$f_2 = 27,568x_1 + 9,786x_2 + 4,048x_3 + 2,889x_4 \quad (14)$$

This is the profit and cost function that we seek to optimized subject to the restrictions imposed on the functional constraints by their limited daily availability.

Furthermore, in any feasible production program, the quantities produced cannot be negative; hence we have the following four inequalities

$$X_i \geq 0, \text{ for } i = 1,\dots,4. \quad (15)$$

The problem can now be stated in mathematical terms as that of finding values of $x_1$, $x_2$, $x_3$ and $x_4$ which satisfy the inequalities (5), (6), (7), (8), (9), (10), (11), (12) and (15) and for which $f_1$ & $f_2$ as given by (13) & (14) are optimized i.e. we seek to determine the values of $x_1$, $x_2$, $x_3$ and $x_4$ so as to efficiently maximize

$f_1 = 14,733x_1 + 1,214x_2 + 4,952x_3 + 3,852x_4$

and minimize

$f_2 = 27,568x_1 + 9,786x_2 + 4,048x_3 + 2,889x_4$

subject to

$1/20x_1+1/100x_2+1/20x_3+1/100x_4 \leq 750$

$60x_1 + 60x_2 + 60x_3 + 60x_4 \leq 1440$

$20x_1 + 15x_2 + 25x_3 + 30x_4m \leq 1440$

$5x_1 + 3x_2 + 3x_3 + 5x_4 \leq 1440$

$x_1 \leq 8,500$, $x_2 \leq 1,234$, $x_3 \leq 1,244$, $x_4 \leq 2,800$

$X_i \geq 0$, for $i = 1, 2, 3, 4$.

We can clearly see that $f_1$ and $f_2$ are not compatible; they are conflicting. Ideally, we would wish to have a solution that both maximize as well as minimize $f_1$ and $f_2$. Unfortunately, such a solution does not exist. But practitioners in the field usually find a number of solutions before deciding on which one is most preferred, Nyiam, (2019).

It is worthy to note here that instead of minimizing the cost function ($f_2$), it is mathematically equivalent to maximizing ($-f_2$). That is:

$$
\left.\begin{array}{ll}
\max f_1 = 14{,}733x_1 + 1{,}214x_2 + 4{,}952x_3 + 3{,}852x_4 & \\
\max f_2 = -27{,}568x_1 - 9{,}786x_2 - 4{,}048x_3 - 2{,}889x_4 & \\
\text{Subject to} & \\
1/20x_1+1/100x_2+1/100x_3+1/20x_4 & \leq 750 \\
60x_1 + 60x_2 + 60x_3 + 60x_4 & \leq 1440 \\
20x_1 + 15x_2 + 25x_3 + 30x_4 & \leq 1440 \\
5x_1 + 3x_2 + 3x_3 + 5x_4 & \leq 1440 \quad (16) \\
x_1 & \leq 8{,}500 \\
x_2 & \leq 1{,}234 \\
x_3 & \leq 1{,}244 \\
x_4 & \leq 2{,}800 \\
X_i \geq 0, \text{ for } i = 1, 2, 3, 4. &
\end{array}\right\}
$$

Problem (16) has all the characteristics of an MOLP problem. In a general MOLP problem, a set of linear objective functions is optimized subject to a set of linear constraints such as (16) above. Therefore, it is a bi-objective MOLP problem called the Niger Mills Company's problem Calabar, Cross River State, Nigeria.

**6.0 Solution of the Problem**

In this section, we present the solution of the Niger Mills Company's problem as returned by our Matlab implementation of the Extended Multi-objective Simplex Algorithm (EMSA) of Nyiam & Salhi (2021) and the interactive MOLP explorer (iMOLPe) of Alves *et al*. (2015) which is an MOLP solver that implements the vector maximization approach (VMA) of Steuer (1986). It is worthy to note here that the VMA is a variant of the MSA that returns all the solutions of the problem.

This problem has two (2) conflicting objective functions, twelve (12) decision variables (including slacks) and eight (8) functional constraints. We reiterate here that a number of efficient solutions were returned by EMSA, but we are only concern here with the nondominated points (objective values) which would be of great importance to the DM as noted in Dauer (1987); Dauer & Liu (1990); Benson (1998a, 1998b & 1998c), Shao & Ehrgott (2008a & 2008b), Ehrgott *et al*. (2011), Ehrgott *et al*. (2012) and Lohne *et al*. (2014) that in practice, the DM prefers to base their choice of a most preferred solution on the nondominated points rather than on the efficient solutions.

The nondominated points returned by EMSA are $f^1 = (f_1^1, f_2^1) = (0 \;\; 0)^T$, $f^2 = (f_1^2, f_2^2) = (353592, -661632)^T$, $f^3 = (f_1^3, f_2^3) = (118848, -97152)^T$ and $f^4 = (f_1^4, f_2^4) = (92448, -69336)^T$ where $f^1 = (f_1^1,\, f_2^1)^T, \ldots,\; f^4 = (f_1^4,\, f_2^4)^T \in Y_N$. This algorithm is designed to return all nondominated points of the problem devoid of redundant ones.

Table 1 Below also shows the iMOLPe output summary which implements the Vector Maximation Approach (VMA) of Steuer (1986) that also returned all the values of the objective functions.

***6.1 The iMOLPe Solver output summary for the Niger Mills Company's Problem***

The second and third columns (i.e under $f_1$ & $f_2$) in Table 1 above are the four (4) set of nondominated points (objective values) returned by iMOLPe. These values confirm what was obtained earlier with our Matlab implementation of EMSA. The values under the fourth column ($x_1$) through the seventh column ($x_4$) are the values of the decision variables; while those values under column eight ($s_1$) through column fifteen ($s_8$) are the values of the slacks variables. The slack variables are all positive at the final solution which means that the constraints are all satisfied and the resources (raw material etc) are in abundant.

**Table 1:** .

| Variable | Row 1 | Row 2 | Row 3 | Row 4 | Row 5 |
|---|---|---|---|---|---|
| **S/N** | 1 | 2 | 3 | 4 | 5 |
| **f1** | 353592 | 0 | | 118848 | 92448 |
| **f2** | -661632 | 0 | | -97152 | -69336 |
| **x1** | 24 | 0 | | 0 | 0 |
| **x2** | 0 | 0 | | 0 | 0 |
| **x3** | 0 | 0 | | 24 | 0 |
| **x4** | 0 | 0 | | 0 | 24 |
| **RHS** | 748.8 | 750 | | 749.76 | 748.8 |
| **s1** | 0 | 1440 | | 0 | 0 |
| **s2** | 960 | 1440 | | 840 | 720 |
| **s3** | 1320 | 1440 | | 1368 | 1320 |
| **s4** | 8476 | 8500 | | 8500 | 8500 |
| **s5** | 1234 | 1234 | | 1234 | 1234 |
| **s6** | 1244 | 1244 | | 1244 | 1244 |
| **s7** | 2800 | 2800 | | 2776 | |

**7.0 Discussion of Result**

From Section 8 above, it can be seen that EMSA of Nyiam & Salhi (2021) and iMOLP$e$ of Alves *et al*. (2015) returns the same set of four (4) non-dominated points. These are

$f^1 = (f_1^1, f_2^1) = (353592, -661632)^T$, $f^2 = (f_1^2, f_2^2) = (0 \ \ 0)^T$, $f^3 = (f_1^3, f_2^3) = (118848, -97152)^T$ and $f^4 = (f_1^4, f_2^4) = (92448, -69336)^T$.

It can be seen clearly that $f_1$, $f_3$ and $f_4$ dominates $f_2$. But there is no clear dominance between $f_1$, $f_3$ and $f_4$ because the value 353,592 is greater than 118848 and 92448, but -661,632 is smaller than -97152 and -69,336. No rational DM would choose the nondominated point $f_2$ which can be deleted from consideration.

The nondominated point $f_1$ implies that a maximum daily profit of ₦353,592.00 would be achieve by Niger Mills Company Plc, Calabar from the sale of her four products namely flour ($x_1$), semovita ($x_2$), hole wheat ($x_3$) and wheat offal ($x_4$) with a minimum total production cost of ₦661,632.00; the nondominated point $f_3$ implies that a maximum daily profit of ₦118,848.00 would be made by the Niger Mills Company, Calabar, with a minimum total production cost of ₦97,152.00 and finally; the nondominated point $f_4$ implies that a maximum daily profit of ₦92,592.00 would be achieve by Niger Mills Company, Calabar with a minimum production cost of ₦69,336.00. We hereby suggest the three production plans for the Company's implementation. The decision to implement or choose the most preferred production plan rest solely on the management of Niger Mills Company, Calabar and it is beyond the scope of this work. This is an area for future research, though we have earlier presented a procedure for determining the most preferred nondominated point of the problem in Nyiam & Salhi (2019).

## 8. Conclusion

We have reviewed the existing literature on the multi-objective simplex Algorithm (MSA), extended multi-objective simplex Algorithm (EMSA) as well as the application of MOLP models to real-world problems. We have also identified and formulated the Niger Mills Company's problem as an MOLP problem. In addition, we have implemented the extended multi-objective simplex algorithm in Matlab and a set of four (4) nondominated points devoid of redundant ones were returned as solutions to the problem. This solution was confirmed by the interactive MOLP explorer (iMOLPe) which also returned the same nondominated points; and finally, we have suggested three different production plans for the Company's implementation while one nondominated point was recommended for deletion from being considered.

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**Statement and Declarations**

**Competing Interests**

The authors declared no competing interests

**Ethical Consideration**

Not applicable

**Funding**

The authors declared ESRC, Grant ES/L011859/1

**Availability of data and material**

All data used in this manuscript are original except those duly acknowledged and cited by the authors.

## Author Contribution

Paschal Bisong Nyiam and Abdellah Salhi jointly contributed to the conception, methodology, analysis, interpretation of results, manuscript preparation, review, and approval of the final version of the manuscript.