

The Effects of Different Activity Duration Distributions in PERT Analysis

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Abstract: Project Evaluation and Review Technique (PERT) has undergone several extensions over more than six decades, particularly in the choice of probability distributions used to represent activity durations. However, the extent to which alternative duration distributions influence PERT estimates remains debated. This study investigated the effects of different activity-duration distributions on PERT estimates using a shipbuilding project network comprising 18 activities and three-point judgmental estimates (optimistic, most likely, and pessimistic durations). Four approaches—PERT-Burr XII, PERT-Beta, Shankar and Sireesha, and Golenko-Ginzburg—were compared based on estimated activity means, variances, coefficients of variation, and project completion time (PCT). For the activity means, the average absolute percentage errors relative to the standardized Beta reference means were 1.6354%, 1.6224%, 1.0938%, and 1.9166% for PERT-Burr XII, PERT-Beta, Shankar and Sireesha, and Golenko-Ginzburg, respectively. Thus, the Shankar and Sireesha method produced the closest mean estimates to the reference values, whereas Golenko-Ginzburg produced the largest average deviation. PERT-Burr XII generally produced higher variance and coefficient-of-variation estimates than the other approaches. Despite these differences at the activity level, the effect on overall project completion time was small. The estimated PCTs were approximately 149 days for PERT-Burr XII, PERT-Beta, and Golenko-Ginzburg, and 150 days for Shankar and Sireesha, compared with approximately 151 days for the standardized Beta reference case. The findings indicate that alternative activity-duration distributions can produce measurable differences in activity-level

parameter estimates but have a relatively limited effect on overall project completion time for the project network examined. Further studies using multiple project networks and independently observed activity-duration data are recommended to establish the generalizability of these findings.

Keywords: *PERT, Activity duration, Activity Distributions, Project Completion Time*

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1.0 Introduction

The Project Evaluation and Review Technique (PERT) is a widely used probabilistic project scheduling technique for estimating project completion time under activity-duration uncertainty (Khan et al., 2023; Kehinde et al., 2017; Iheonu and Achom, 2023; Farida and Anenda, 2022). Developed in the late 1950s, PERT was designed to support the planning and scheduling of projects in which activity durations are uncertain and difficult to predict precisely.

The original PERT formulation assumes that activity durations follow a Beta distribution, with the mean and variance ($\mu = (a + 4m + b)/6$ and $\sigma^2 = (b - a)^2/36$ respectively) estimated from three judgmental time estimates: optimistic, most-likely, and pessimistic durations (MacCrimmon and Ryavec, 1964). The resulting estimates of activity mean and variance are subsequently

used to estimate the expected project completion time and its associated uncertainty. However, the Beta distribution used in conventional PERT has been criticized as restrictive because its bounded support and relatively light tails may not adequately represent activity durations characterized by substantial positive skewness or extreme values. Real-world project activities may be affected by delays, resource constraints, technical difficulties, and other sources of uncertainty, which can result in skewed or heavy-tailed activity-duration distributions (Lopez-Martin et al., 2012; Udoumoh et al., 2017). Consequently, several extensions and modifications of PERT have been proposed to accommodate activity durations with longer or heavier tails and to provide more flexible representations of duration uncertainty (Hahn, 2008; Hahn and Martin, 2015; Udoumoh and Iwok, 2025; Sackey and Kim, 2019; Mohan et al., 2007; McCombs et al., 2008).. Udoumoh and Ebong (2017) reviewed several probability distributions proposed for modelling activity durations in PERT and highlighted the continuing development of alternatives to the conventional Beta-based formulation. Although these alternative distributions provide greater flexibility in representing activity-duration behaviour, their practical importance depends on whether the choice of distribution produces materially different estimates of activity characteristics and project completion time.

Udoumoh et al. (2017) reported that the activity durations of a borehole-drilling project were approximately Burr XII distributed. . Subsequently, Udoumoh and Iwok (2025) proposed a method for estimating the parameters of the Burr XII distribution within the PERT framework. PERT-Burr XII was reported to produce more accurate estimates of activity means than some existing PERT-based methods

This issue is particularly important in practical PERT applications because historical activity-

duration data are often unavailable or insufficient to establish the underlying empirical probability distribution through goodness-of-fit analysis.. An answer to this question is necessary because in most practical cases, project data are often not handy to enable goodness-of-fit test in order to determine the shape of the empirical distribution. That is why in application, PERT rely heavily on judgmental activity duration estimates. This raises an important question concerning how the use of different, potentially more flexible distributional forms influences PERT estimates when the same three-point judgmental estimates are used

Although several alternative activity-duration distributions have been proposed, there is limited empirical evidence on the extent to which the choice of distribution affects estimates of activity means, variances, and, most importantly, project completion time when only three-point judgmental estimates are available. In particular, it remains unclear whether differences in distributional assumptions observed at the activity level translate into meaningful differences in estimated project completion time. (Bianchi, & Walker, 2026; Cao& Schei behenne, 2025; Reshi, 2023).

The literature provides differing views regarding the practical importance of selecting alternative activity-duration distributions in PERT. While some researchers have advocated the use of more flexible and realistic probability distributions for modelling activity durations, others have questioned whether introducing additional distributions provides meaningful improvements in PERT estimates (Clark, 1962). (Clark, 1962). Hajdu and Bokor (2014) argued that the choice of activity-duration distribution may have little effect on PERT estimates compared with the accuracy of the underlying judgmental estimates “Resolving this issue is important because it can establish whether the additional complexity associated with alternative activity-



duration distributions is justified in practical PERT applications

The findings of such an assessment can provide useful guidance to project planners and researchers in determining whether the selection of an activity-duration distribution materially influences PERT estimates. If different distributions produce comparable project completion-time estimates, practitioners may have limited need to adopt more complex distributional models when only judgmental estimates are available. Conversely, substantial differences would indicate that greater attention should be given to the choice of distribution when applying PERT.

The aim of this study is to evaluate the effect of different activity-duration distributions on PERT estimates of activity means, variances, and project completion time. Specifically, the study compares four approaches: PERT-Beta, PERT-Burr XII, the Shankar and Sireesha modification of PERT-Beta, and the Golenko-Ginzburg modification of PERT-Beta, using project-network data from the shipbuilding domain reported by Sireesha and Shankar (2011)

The four approaches are evaluated using the same three-point judgmental activity-duration estimates, with comparisons based on their estimates of activity means and variances and the resulting project completion time. By determining whether alternative distributional assumptions produce materially different PERT estimates, the study provides empirical evidence on the robustness of PERT to the choice of activity-duration distribution and contributes to the ongoing debate on the practical value of extending the conventional PERT framework.

2.0 Methods

2.1 PERT-Beta Method

The study evaluates four approaches for estimating activity-duration characteristics within the PERT framework: PERT-Beta, Golenko-Ginzburg modification of PERT-

Beta, Shankar and Sireesha modification of PERT-Beta, and PERT-Burr XII. The approaches are applied to the same three-point activity-duration estimates, and their performance is compared in terms of estimated activity means, variances, and project completion time

The original PERT formulation developed by Malcolm et al. (1959) assumes that activity durations follow a generalized Beta distribution, whose probability density function is given in equation 1. $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{(b-a)^{\alpha+\beta-1}} ; a < x < b, \alpha, \beta > 0$ (1)

where α and β are the shape parameters and $\Gamma(.)$ is the gamma function.

In conventional PERT, the optimistic (aa), most-likely (mm), and pessimistic (bb) estimates define the lower bound, modal value, and upper bound, respectively, of the activity-duration distribution.

The expected activity duration and variance in the conventional PERT-Beta approach are estimated from the three-point judgmental estimates using equations 2 and 3, respectively.

$$\hat{\mu}_x = \frac{(a+4m+b)}{6} \quad (2)$$

$$\hat{\sigma}_x^2 = \frac{(b-a)^2}{36} \quad (3)$$

These estimates are subsequently used as the activity-duration parameters for the project network analysis.

Farnum and Stanton (1997) showed that the mean of Equation (2) is appropriate within some range of modal values: $a + 0.13(b - a) < m < b - 0.13(b - a)$. When the most-likely estimate is located within approximately 13% of either extreme of the activity-duration range, the conventional PERT-Beta mean may provide a poor approximation. This limitation provides a motivation for examining alternative distributional formulations that may provide more appropriate estimates when activity durations exhibit greater skewness or longer tails.



2.2 Gollenko-Ginzburg modifications on PERT-Beta

Golenko-Ginzburg (1988) proposed a modification of the PERT-Beta method in which the mean and variance of the generalized Beta distribution in Equation (1) are estimated using the expressions given below. $\hat{\mu}_x = \frac{(2a+9m+2b)}{13}$ (4)

$$\hat{\sigma}_x^2 = \frac{(b-a)^2}{1268} \left[22 + 81 \frac{m-a}{b-a} - 81 \left(\frac{m-a}{b-a} \right)^2 \right] \quad (5)$$

Equations (4) and (5) were further improved upon to accommodate longer-tailed activity durations with the mean and variance given as Equations (6) and (7)

$$\hat{\mu}_x = \frac{(8a+5b)}{13} \quad (6)$$

$$\hat{\sigma}_x^2 = \frac{10(b-a)^2}{317} \quad (7)$$

2.3 Shankar & Sireesha modification on PERT-Beta

Shankar and Sireesha (2009) modified the Golenko-Ginzburg expressions by transforming Equations (4) and (6) into Equations (10) and (11), respectively, to accommodate activity durations with longer tails.

$$\hat{\mu}_x = \frac{(5a+17m+5b)}{27} \quad (8)$$

$$\hat{\sigma}_x^2 = \frac{(17m-27a+10b)(27b-10a-17m)}{2300} \quad (9)$$

Substituting $m = \frac{2a+b}{3}$ in Equation (9) yielded equation 10.

$$\hat{\sigma}_x^2 = \frac{(b-a)^2}{35} \quad (10)$$

Equation 10 was subsequently used to obtain the corresponding activity-duration estimate under the Shankar and Sireesha modification.

Solving Equations (15-17) simultaneously yielded the following expressions

$$c = \ln \left(\frac{\frac{\ln R(x_b)}{\ln R(x_a)} - 1}{\frac{\ln R(x_m)}{\ln R(x_a)} - 1} \right) / \ln \left(\frac{x_m}{x_b} \right) \quad (18)$$

$$k = \frac{\ln R(x_a)}{\ln(e^Q + 1)} \quad (19)$$

2.4 PERT-Burr XII

Udoumoh & Iwok (2025) proposed the use of the three-parameter Burr XII distribution to approximate activity-duration distributions within the PERT framework. The probability density function of the Burr XII distribution is given in equation 11

$$f(x) = ck \frac{x^{c-1}}{\alpha^c} \left(1 + \left(\frac{x}{\alpha} \right)^c \right)^{-(k+1)}; x \geq 0, c > 0, k > 0, \alpha > 0 \quad (11)$$

The cumulative density function is

$$F(x) = 1 - \left(1 + \left(\frac{x}{\alpha} \right)^c \right)^{-k} \quad (12)$$

The mean and variance are given as Equations (13) and (14).

$$E(X) = \frac{k\alpha\Gamma(k-1/c)\Gamma(1/c+1)}{\Gamma(k+1)}; ck > 1 \quad (13)$$

$$V(X) = \frac{k\alpha^2\Gamma(k-2/c)\Gamma(2/c+1)}{\Gamma(k+1)} - \left(\frac{k\alpha\Gamma(k-1/c)\Gamma(1/c+1)}{\Gamma(k+1)} \right)^2 \quad (14)$$

The parameters of the Burr XII distribution were estimated via quantile estimation where x_a , x_m and x_b represented the lower, median and upper expert judgment percentiles of a 3-parameter Burr XII activity duration with scale parameter α , and shape parameters c and k . Consequently, considering Equation (12), the lower, median and upper quantiles of a 3 Parameter Burr XII distribution were given as Equations (15-17), respectively.

$$x_a = \alpha \left[\left(1 - F(x_a) \right)^{-\frac{1}{k}} - 1 \right]^{\frac{1}{c}} \quad (15)$$

$$x_m = \alpha \left[\left(1 - F(x_m) \right)^{-\frac{1}{k}} - 1 \right]^{\frac{1}{c}} \quad (16)$$

$$x_b = \alpha \left[\left(1 - F(x_b) \right)^{-\frac{1}{k}} - 1 \right]^{\frac{1}{c}} \quad (17)$$



$$\alpha = \frac{1}{x_a} \left\{ \ln \left[\left(e^{Q+1} \frac{\ln R(x_b)}{\ln R(x_a)} - 1 \right) \ln \left(\frac{x_m}{x_b} \right) - \ln \left(\frac{x_b}{x_a} \right) \ln \left[\left(e^{Q+1} \frac{\ln R(x_m)}{\ln R(x_a)} - 1 \right) + \ln \left(\frac{x_b}{x_a} \right) \ln \left[\left(e^{Q+1} \frac{\ln R(x_b)}{\ln R(x_a)} - 1 \right) \right] \right] \right\} / e^{\left\{ \ln \left[\left(e^{Q+1} \frac{\ln R(x_b)}{\ln R(x_a)} - 1 \right) \right] \right\}}$$

(20)

where Q is the root of

$$x \ln \left(\frac{x_m}{x_b} \right) - \ln \left(e^{\frac{\ln R(x_b) \ln(e^x + 1)}{\ln R(x_a)}} - 1 \right) \ln \left(\frac{x_m}{x_b} \right) + \ln \left(\frac{x_b}{x_a} \right) \ln \left(e^{\frac{\ln R(x_m) \ln(e^x + 1)}{\ln R(x_a)}} - 1 \right) - \ln \left(\frac{x_b}{x_a} \right) \ln \left(e^{\frac{\ln R(x_b) \ln(e^x + 1)}{\ln R(x_a)}} - 1 \right) = 0$$

(21)

$F(x_a)$ or $R(x_a)$, and $F(x_b)$ or $R(x_b)$ for the computation of c , k and α .

2.5 Comparison of the PERT Methods

The four PERT approaches produced activity-mean estimates, which were compared using the absolute percentage error (APE), following Sireesha and Shankar (2011) and Mohan et al. (2007). For this comparison, the standardized activity means were treated as reference or true values, while the means obtained from the respective PERT approximations were treated as estimated values.

$$\text{Absolute Percentage Error} = \left| \frac{\text{Estimated Value} - \text{True Value}}{\text{True Value}} \right| \times 100 \quad (22)$$

A smaller APE indicates greater agreement between the estimated and reference activity means, whereas a larger APE indicates greater deviation from the reference value.

The effect of the different activity-duration distributions on project completion time (PCT) was subsequently assessed using the Critical Path Method (CPM). For each PERT approach, the estimated activity durations were assigned to the corresponding activities in the project network, and the resulting critical path was identified. The total duration of the critical path was taken as the estimated project completion time.

2.6 The Data

Data from a project network in the shipbuilding domain, as presented in Sireesha and Shankar (2011), was considered for the study (See

Appendix A for the data). The dataset contains three judgmental estimates for each activity: aa, the optimistic duration; mm, the most-likely duration; and bb, the pessimistic duration, together with the corresponding skewness coefficients. Here, activity duration is assumed to follow the generalised beta distribution. transforming using $y = a + (b - a)x$. The transformation yielded the results in Equations (25) and (26) as in Mohan, *et al.* (2007).

$$\mu = a + (b - a) * \alpha / (\alpha + \beta) \quad (25)$$

$$\sigma^2 = (b - a)^2 * \alpha \beta / [(\alpha + \beta + 1)(\alpha + \beta)^2] \quad (26)$$

To obtain the three quantiles required for estimating the Burr XII parameters, 1,000 random activity-duration observations were generated for each activity using the PERT-Beta distribution based on the corresponding optimistic (aa), most-likely (mm), and pessimistic (bb) estimates in Appendix A. The 5th, 50th, and 95th percentiles of the simulated observations were then calculated and used as the lower, median, and upper quantile inputs in Equations (20)–(23) for estimating the Burr XII parameters.

Given the 1000 random sample, the 5th, 50th, and 95th percentiles were then computed, and the results were then substituted into Equations (20)–(23) to estimate Burr XII parameters. Using the estimated parameters, the mean and



variance in Equations (13) and (14) were easily computed. The analysis was carried out with the help of Maple and TORA optimisation software.

3.0 Results and Discussion

Table 1 presents the estimated mean activity durations obtained using the PERT-Burr XII, PERT-Beta, Shankar & Sireesha, & Golenko-Ginzburg methods, together with the corresponding standardized Beta means used as reference values. . The estimated activity means are numerically close across the four methods, although the magnitude of their deviations from the standardized reference means varies among activities.. However, comparison of the absolute percentage errors in Table 2 provides a more informative

assessment of the relative accuracy of the four methods.. Table 2 shows that the absolute percentage errors of the activity-mean estimates obtained using PERT-Burr XII are generally lower than those obtained using PERT-Beta and Golenko-Ginzburg. However, the Shankar and Sireesha method produces smaller absolute percentage errors than PERT-Burr XII for several activities. Thus, no single method produces the smallest error for every activity, indicating that the relative accuracy of the methods varies across the project network. However, a formal correlation or regression analysis between the absolute percentage errors and skewness coefficients would be required to establish whether a statistically meaningful relationship exists. .

Table 1: Mean Estimates of Activity Duration

Activity	PERT-Burr XII	PERT-Beta	Shankar & Sireesha	Gollenko-Ginzburg	Standardized Mean
1-2	25.3420	25.3333	25.3704	25.3079	25.52
1-3	26.3420	26.3333	26.3704	26.3077	26.52
1-8	28.3420	28.3333	28.3704	28.3077	28.52
1-5	22.836	22.6667	22.7407	22.6154	22.99
2-7	37.6777	37.6667	37.7407	37.6154	37.95
3-4	18.6780	18.6667	18.7407	18.6154	18.95
8-9	35.4290	35.5000	35.5556	35.4615	35.73o
5-6	5.6899	5.6667	5.7407	5.6154	5.85
4-6	14.4140	14.5000	14.5556	14.4615	14.76
6-7	7.6882	7.6667	7.7407	7.6154	7.85
7-9	30.5200	30.6667	30.7407	30.6154	30.99
9-10	7.4824	7.5000	7.5556	7.4615	7.67
10-11	7.6882	7.6067	7.7407	7.6154	7.85
10-12	5.4832	5.5000	5.5556	5.4615	5.67
11-14	20.4120	20.5000	20.5556	20.4615	20.76
12-13	7.3393	7.3333	7.3704	7.3077	7.47
13-14	10.6870	10.6667	10.7407	10.6154	10.85
14-15	15.4300	15.5000	15.5556	15.4615	15.73

The observed absolute percentage errors do not exhibit an obvious monotonic relationship with the reported skewness coefficients. This suggests that the magnitude of the estimation errors may not be explained solely by the tail characteristics or skewness of the activity-

duration distributions. Across the 18 activities, the reported average absolute percentage errors for the mean estimates are 1.6354% for PERT-Burr XII, 1.6224% for PERT-Beta, 1.0938% for Shankar and Sireesha, and 1.9166% for Golenko-Ginzburg.



. Based on the average absolute percentage error, the Shankar and Sireesha method ranks first, followed by PERT-Beta, PERT-Burr XII, and Golenko-Ginzburg, respectively. Thus,

among the four approaches, the Shankar and Sireesha method produces the closest activity-mean estimates to the standardized Beta reference means on average

Table 2: Absolute Percentage Error of Mean Estimates

Activity	PERT-Burr XII	PERT-Beta	Shankar & Sireesha	Gollenko-Ginzburg
1-2	0.6974	0.7316	0.5862	0.8311
1-3	0.6712	0.704	0.5641	0.8005
1-8	0.6241	0.6546	0.5245	0.7444
1-5	0.6699	1.4063	1.0844	1.6294
2-7	0.7175	0.7465	0.5515	0.8817
3-4	1.4353	1.4950	1.1045	1.7657
8-9	0.8424	0.6437	0.4881	0.7515
5-6	2.7367	3.1333	1.8684	4.0103
4-6	2.3442	1.7615	1.3848	2.0224
6-7	2.0612	2.3350	1.3924	2.9885
7-9	1.5166	1.0432	0.8045	1.2088
9-10	2.4459	2.2164	1.4915	2.7184
10-11	2.0612	3.0994	1.3924	2.9885
10-12	3.2945	2.9982	2.0176	3.6773
11-14	1.6763	1.2524	0.9845	1.4379
12-13	1.7497	1.8300	1.3333	2.1727
13-14	1.5023	1.6894	1.0074	2.1622
14-15	1.9072	1.4622	1.1087	1.7069

Although the ranking differs among the methods, the average absolute percentage errors are relatively small, ranging from 1.0938% to 1.9166%. The difference between the best-performing method (Shankar and Sireesha) and the least accurate method (Golenko-Ginzburg) is less than one percentage point. This indicates that, for the dataset examined, the choice among the four approaches has a relatively modest effect on estimates of activity means.

Table 3 presents the estimated variances of the 18 activity durations obtained using PERT-Burr XII, PERT-Beta, Shankar and Sireesha, and Golenko-Ginzburg. The results show that the variance estimates obtained using PERT-Beta, Shankar and Sireesha, and Golenko-Ginzburg are generally lower than those obtained using PERT-Burr XII.”. is an

indication that these methods perform better than PERT-Burr XII in terms of precision. To further compare the relative variability of the activity-duration estimates, the coefficient of variation (CV) was calculated for each method.. able 4 presents the coefficients of variation for the 18 activities under the four PERT approachesTable 4 shows that the coefficients of variation for PERT-Beta, Shankar and Sireesha, and Golenko-Ginzburg are generally lower than those for PERT-Burr XII. The lower coefficients of variation indicate that these three methods produce estimates with lower relative variability than PERT-Burr XII for the activities considered.

Although PERT-Burr XII generally produces the largest coefficients of variation, the differences among the four approaches are relatively small for several activities. The



ranking also varies across activities, indicating that no single method consistently produces the lowest relative variability for every activity

Table 3: Variance of Activity Durations

Activity	PERT-Burr XII	PERT-Beta	Shankar & Sireesha	Gollenko-Ginzburg	Standardized Variance
1-2	2.3076	1.7778	1.8286	2.0686	3.02
1-3	2.3071	1.7778	1.8286	2.0686	3.02
1-8	2.3063	1.7778	1.8286	2.0686	3.02
1-5	3.4736	2.7778	2.8571	3.0765	4.43
2-7	2.08	1.7778	1.8286	1.8770	2.68
3-4	2.0943	1.7778	1.8286	1.8770	2.68
8-9	1.7532	1.3611	1.4000	1.4889	2.14
5-6	1.3091	1.0000	1.0286	0.9440	1.33
4-6	3.0285	2.2500	2.3140	2.5552	3.7
6-7	1.2764	1.0000	1.0286	0.9440	1.33
7-9	3.6331	2.7778	2.8571	3.0765	4.43
9-10	0.7549	0.6944	0.7143	0.6893	0.98
10-11	1.2764	1.0000	1.0286	0.9440	1.33
10-12	0.7624	0.6944	0.7143	0.6893	0.98
11-14	3.0116	2.2500	2.3143	2.5552	3.7
12-13	0.5252	0.4444	0.4571	0.4692	0.67
13-14	1.2502	1.0000	1.0286	0.9440	1.33
14-15	1.78	1.3611	1.4000	1.4890	2.14

Table 4: Coefficient of Variation (in Percentages)

Activity	PERT-Burr XII	PERT-Beta	Shankar & Sireesha	Gollenko-Ginzburg
1-2	5.99	5.26	5.33	5.68
1-3	5.77	5.06	5.13	5.47
1-8	5.36	4.71	4.77	5.08
1-5	8.16	7.35	7.43	7.76
2-7	3.83	3.54	3.58	3.64
3-4	7.75	7.14	7.22	7.36
8-9	3.74	3.29	3.33	3.44
5-6	20.11	17.65	17.67	17.30
4-6	12.07	10.34	10.45	11.05
6-7	14.69	13.04	13.10	12.76
7-9	6.25	5.43	5.50	5.73
9-10	11.61	11.11	11.19	11.13
10-11	14.69	13.15	13.10	12.76
10-12	15.92	15.15	15.21	15.20



11-14	8.50	7.32	7.40	7.81
12-13	9.87	9.09	9.17	9.37
13-14	10.46	9.37	9.44	9.15
14-15	8.65	7.53	7.61	7.89

Project Completion Time (PCT)

“Using the mean activity-duration estimates in Table 1, the project completion time (PCT) was determined for each PERT approach and for the standardized Beta reference case using the Critical Path Method (CPM). The results are presented in Figure 1. Figure 1 shows that the estimated PCT is approximately 149 days for PERT-Burr XII, PERT-Beta, and Golenko-Ginzburg, compared with approximately 150 days for the Shankar and Sireesha method. This finding is consistent with Hajdu and Bokor (2014), who reported that using different activity-duration distributions with the same three-point judgmental estimates may have only a limited effect on estimated project completion time. The difference of approximately one day among the four estimates suggests that the distributional and parameter-estimation differences that produce observable changes at the activity level are largely attenuated when the activity durations are aggregated along the project network.

The standardized Beta reference case produces a PCT of approximately 151 days, which is higher than the estimates obtained from the

four approximation methods.. Because the standardized Beta reference PCT is higher than the estimates produced by the four approximation methods, the latter methods yield lower PCT estimates for this particular project network. However, this difference should not automatically be interpreted as systematic underestimation because the standardized Beta case represents the reference model used in this study rather than an independently observed project completion time (Elmaghraby, 1977; Ballesteros-Perez, 2017).

The difference between the standardized Beta estimate and the approximate methods is approximately 1–2 days, indicating that the choice of activity-duration approximation has a relatively small effect on the overall PCT for the project considered.

Fig. 1 compares the project completion times obtained from the four PERT approaches with the standardized Beta reference case, thereby showing whether differences observed in activity-level estimates propagate to the overall project completion time.



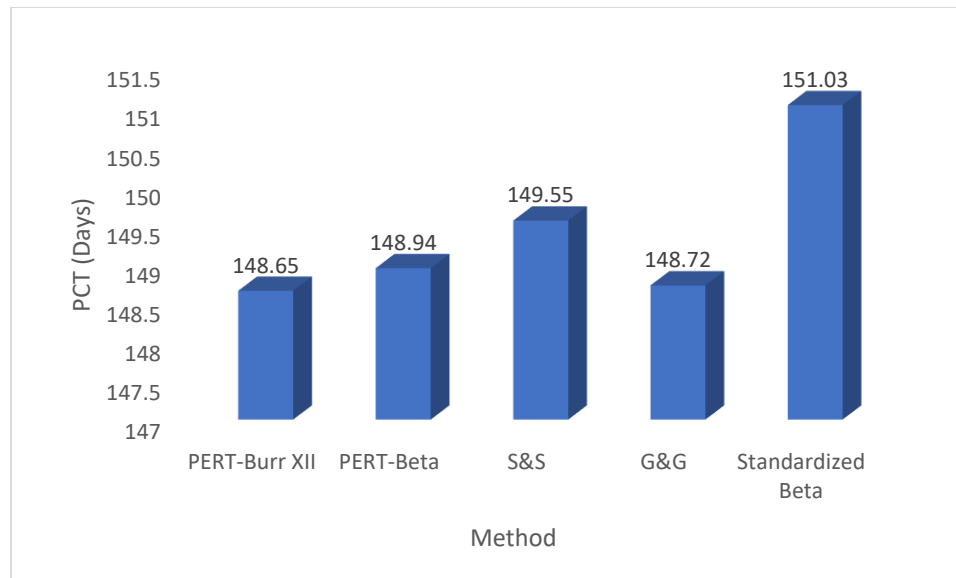


Fig. 1: Project Completion Time Estimates from the Four PERT Methods and the Standardized Beta Reference Case

Taken together, the results demonstrate an important distinction between activity-level and project-level effects. The four approaches produce measurable differences in estimated activity means, variances, and coefficients of variation, as shown in Tables 1–4. However, these differences translate into only small changes in the overall project completion time, with estimates clustered around 149–150 days compared with approximately 151 days for the standardized Beta reference case. This suggests that, for the shipbuilding project examined, differences in the assumed activity-duration distribution have a limited effect on the final PCT despite producing observable differences in individual activity estimates

4.0 Conclusion

This study examined the effects of alternative activity-duration distributions and approximation methods on PERT estimates using the same three-point judgmental activity-duration estimates. Specifically, PERT-Burr XII, PERT-Beta, Shankar and Sireesha, and Golenko-Ginzburg approaches were compared in terms of estimated activity means, variances, coefficients of variation, and project completion time. The results showed that the choice of distribution and estimation approach

produces measurable differences in activity-level parameter estimates. Based on the average absolute percentage error of the estimated activity means, the Shankar and Sireesha method produced the closest estimates to the standardized Beta reference means, followed by PERT-Beta, PERT-Burr XII, and Golenko-Ginzburg. The results also showed that PERT-Burr XII generally produced higher variance and coefficient-of-variation estimates than the other approaches.

Despite these differences at the activity level, the effect on overall project completion time was relatively small. The estimated project completion times for PERT-Burr XII, PERT-Beta, Golenko-Ginzburg, and Shankar and Sireesha were approximately 149, 149, 149, and 150 days, respectively, compared with approximately 151 days for the standardized Beta reference case. Thus, for the shipbuilding project network examined, the choice of activity-duration distribution and associated approximation method had a limited effect on the overall project completion time, even though it affected individual activity parameter estimates. The findings therefore suggest that introducing alternative activity-duration distributions into PERT may provide greater



differentiation at the activity-parameter level than at the aggregate project-completion level. Further empirical studies using multiple project networks, independently observed activity-duration data, and different degrees of skewness and tail behaviour are recommended to determine whether the limited effect observed on project completion time is consistent across different project types and network structures.

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Compliance with Ethical Standards Declaration

Ethical Approval

Not Applicable

Competing interests

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Author's Contribution

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Appendix A: Project Network on Ship Building

Activity	Activity Description	<i>a</i>	<i>m</i>	<i>b</i>	Skewness
1-2	Shell:Loft	22	25	30	1.7
1-3	I.B. Structure: Layout	23	26	31	1.7
1-8	Mach Fdn:Loft	25	28	33	1.7
1-5	I.B. Piping Laying	19	22	29	2.3
2-7	Shell Assemble	35	37	43	3
3-4	I. B. Structure:Fab	16	18	24	3
8-9	Mach Fdn: Fabricate	33	35	40	2.5
5-6	I.B. piping Fab.	4	5	10	5
4-6	I.B. Structure:Assembly	11	14	20	2
6-7	I.B.Structure: Install	6	7	12	5
7-9	Erect I. B.	27	30	37	2.5
9-10	Erect Foundation	6	7	11	4
10-11	Engine: Install	6	7	12	5
10-12	Complete #rd DK	4	5	9	4
11-14	Engine:Finish	17	20	26	2
12-13	Boiler:Install	6	7	10	3
13-14	Boiler Test	9	10	15	5
14-15	Final Test	13	15	20	2.5

